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Fundamentals of Internal Model Control

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Introduction

In the process industries -- be it refining, chemical, pulp and paper, or power -- process control is usually realized through a control hierarchy that is composed of two layers. The upper layer, often called advanced control, is used to calculate set points for the lower or regulatory control layer [1]. With this control structure, the upper layer is typically a multivariable controller that is responsible for meeting the control objectives set for the entire process. On the other hand, regulatory control loops are usually concerned with the control of a single, fundamental process variable, such as flow, pressure, temperature, composition, or level. However, effective multivariable process control requires well designed and well behaved regulatory control loops.

This paper reviews the fundamentals of internal model control (IMC) as it applies to stable, single input, single output regulatory control loops. The intent of this paper is to present a pragmatic approach to loop tuning and not to delve too deeply into the nuances of control theory. For those who are interested, well presented extensions to multivariable systems and unstable plants can be found in Doyle et al. [2].

Control of stable plants

Consider the unity gain feedback controller illustrated in Fig. 1 in which *C* represents the transfer function of the controller and *P* represents the transfer function of the plant. For this system, the closed-loop transfer function can be written as

$$\frac{y}{r} = \frac{PC}{1+PC}\,. \tag{1}$$

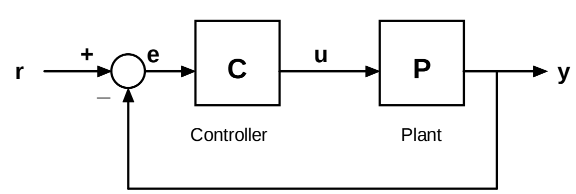

Fig. 1: Generic block diagram of a control system with unity feedback.

One of the difficulties in designing a controller for this system is that the closed-loop transfer function is nonlinear in *C*. However, if we replace the quantity *C / (1 + PC)* with the transfer function *Q* such that

$$\frac{y}{r} = PQ \tag{2}$$

we can see that the closed-loop transfer function is now linear in *Q*. Therefore, given *Q* -- which may be *any* stable transfer function -- we can back-calculate *C* as

$$C = \frac{Q}{1 - PQ}. \qquad (3)$$

This technique of controller design is often referred to as Q-parameterization, Youla parameterization or, in the process industries, as internal model control (because the control law contains a model of the plant). The concepts behind internal model control were introduced in the late 1950s and later popularized by Morari and Zarifiou [3]. Although easily derived, Eq. 3 can provide profound insights into controller design. In particular, Eq. 3 represents the set of all controllers for which the feedback system is internally stable [2]. In other words, for *any* stable plant *P* and *any* stable transfer function *Q*, *C* will be an internally stable controller. Put another way, all internally stable controllers must take the form of Eq. 3.

To implement *C* as described by Eq. 3, we must make one slight modification. In reality, we do not have perfect knowledge of the plant dynamics and we need to replace the transfer function *P* with its estimate $\hat{P}$:

$$\mathrm{C} = \frac{\mathrm{Q}}{1 - \hat{\mathrm{P}}\mathrm{Q}}. \qquad (4)$$

In practice, $\hat{P}$ can be estimated from first principle models or from black-box tests in which a step sequence is applied to the process and a linear model is fit to the experimental data.

A graphical interpretation of the IMC control structure of Eq. 4 is illustrated in Fig. 2. More commonly, however, the IMC control structure is represented by the equivalent block diagram in Fig. 3.

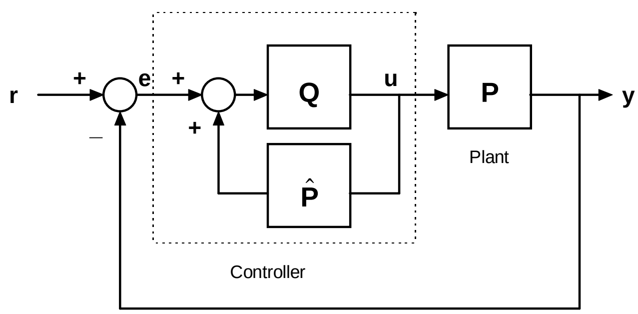

Fig. 2: Block diagram realization of Eq. 4.

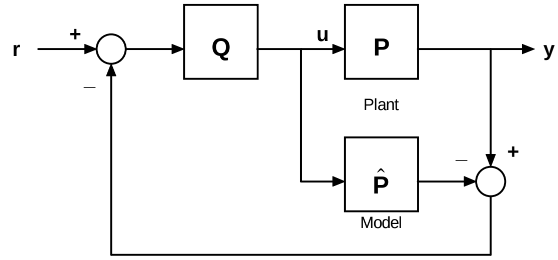

Fig. 3: Equivalent block diagram realization of Eq. 4

Example 1

Many industrial processes have dominant time constants that are much greater than all other system dynamics, including transport lags. For these systems, the process dynamics can be approximated by the first-order model

$$\frac{y}{u} = \frac{K_p}{\tau s + 1}, \tag{5}$$

where K_p is the process gain and τ is the dominant process time constant. For simplicity, we can choose Q as

$$Q = P^{-1} \frac{1}{\lambda s + 1} \tag{6}$$

so that the closed-loop transfer function reduces to

$$PQ = \frac{1}{\lambda s + 1}. \tag{7}$$

In this equation, λ is the desired closed-loop time constant and is usually set between 0.5τ and 5τ for fast processes, such as liquid flow. Substituting Eq. 6 into Eq. 3, we get a standard PI controller of the form

$$C = \frac{\tau}{\lambda K_p}\left(1 + \frac{1}{\tau s}\right). \tag{8}$$

When using internal model control, the selection of Q is arbitrary so long as it is a stable transfer function. Of course, some choices are easier to implement than others. Indeed, by carefully selecting Q, we can reduce the controller to a single tuning parameter. If the plant is stable and does not have any zeros in the right-half plane (i.e., P^{-1} is a stable transfer function), Q can be set to

$$Q = P^{-1} F, \tag{9}$$

where F is the desired closed-loop transfer function, usually chosen to be an n^{th}-order, critically damped system of the form

$$F = \frac{1}{(\lambda s + 1)^n}. \tag{10}$$

In this equation, λ is the closed-loop time constant and is used as the single controller tuning parameter. For parsimony, n is set as small as possible so as to make Q proper (viz., to make the degree of the numerator polynomial less than or equal to the degree of the denominator polynomial).

Alternatively, at the expense of simplicity, additional control objectives can be met by adding a lead to the desired closed-loop transfer function:

$$F = \frac{\beta s + 1}{(\lambda s + 1)^n}, \tag{11}$$

where β is an adjustable tuning parameter that can be used, for example, to improve the load rejection characteristics of the controller [4]. Another common use of the lead function is to design a controller that can track ramp as well as step reference signals. To track a step, we require $\lim_{s \to 0}$ (PQ) = 1, and to track a ramp, we require $\lim_{s \to 0} \frac{d}{ds}$ (PQ) = 0. Combining these boundary conditions with the relation $Q = P^{-1}F$, Eq. 11 becomes

$$F = \frac{n\lambda s + 1}{(\lambda s + 1)^n} . \qquad (12)$$

A word of caution is in order. For a control loop to be able to track a ramp, it must have at least two integrators [2]. A controller based on the desired closed-loop transfer function in Eq. 12 will satisfy this condition; however, in practice, we usually prefer to keep the number of integrators to a minimum since integrators are prone to forming limit cycles in the presence of system nonlinearities.

Control of stable plants with non-minimum phase dynamics

The control law described by Eq. 4 is valid for minimum phase as well as for non-minimum phase systems (i.e., systems with zeros in the right-half plane). However, with non-minimum phase systems, we cannot invert the plant dynamics because this would make *Q* unstable. This, in turn, will make the control loop internally unstable even though the dynamics of the plant and the dynamics of the inverted plant cancel. Consider, for example, the two open-loop systems in Fig. 4. Although the transfer functions from *u* to *y* are stable (and identical), the transfer function from *u* to *x* in the upper block diagram is unstable. As such, the intermediate state *x* will become unbounded -- or, if implemented in hardware, will hit a saturation limit.

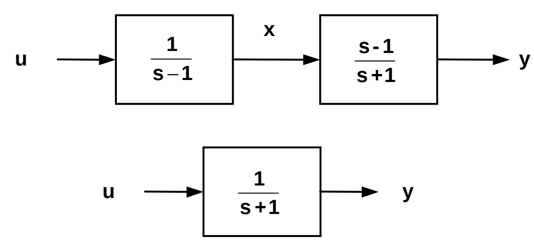

Fig. 4: An example of system dynamics that are not internally equivalent

In short, non-minimum phase dynamics cannot be canceled and must be maintained in the design of the closed-loop transfer function. To emphasize this point, theorists have factored the plant dynamics into minimum phase and non-minimum phase components in the fashion

$$P = P_+ P_- , \qquad (13)$$

where P_+ contains all of the right-half plane zeros and P_- contains all of minimum phase components. With this decomposition, we can select *Q* by inverting the minimum phase dynamics in the usual manner:

$$Q = P_-^{-1} F , \qquad (14)$$

where *F* is defined by Eqs. 10 and 11 and is subject to the usual boundary conditions. After factoring, the closed-loop transfer function reduces to

$$PQ = P_+ F , \qquad (15)$$

which contains all of the non-minimum phase components as required.

Example 2

Continuing with the previous example, suppose we want to refine the first-order model by including an estimate for the process dead time; namely,

$$\frac{y}{u} = \frac{K_p e^{-T_d s}}{\tau s + 1}, \tag{16}$$

where T_d is the process dead time. Provided that $T_d << \tau$, we can approximate the dead time dynamics by a first-order Taylor series expansion:

$$\frac{y}{u} \approx \frac{K_p}{\tau s + 1}(1 - T_d s). \tag{17}$$

Separating the model into minimum phase (P_-) and non-minimum phase (P_+) components we get

$$P_- = \frac{K_p}{\tau s + 1} \quad \text{and} \quad P_+ = (1 - T_d s). \tag{18}$$

In accordance with Eq. 14, choose Q as

$$Q = P_-^{-1} \frac{1}{\lambda s + 1}. \tag{19}$$

Substituting Eqs. 17 and 19 into Eq. 3, we get the PI controller

$$C = \frac{\tau}{K_p(\lambda + T_d)}\left(1 + \frac{1}{\tau s}\right), \tag{20}$$

which is simply a reduced-gain version of the controller derived for the system without dead time. Alternatively, gains for a full PID controller can obtained by replacing the Taylor series approximation in Eq. 17 with a first-order Pade approximation.

IMC Load Disturbance Rejection Characteristics

So far, we have been concerned with designing controllers about set point changes. However, attention should be given to the controller's load disturbance rejection characteristics, which may differ significantly from those associated with set point changes. Take, for example, the system illustrated in Fig. 5 in which a load disturbance is introduced upstream of the plant dynamics.

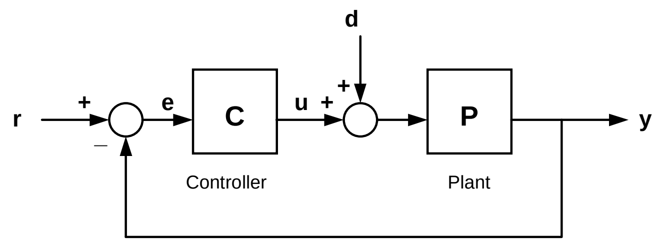

Fig. 5: Generic block diagram of a control system with a load disturbance

For this system, the complete, closed-loop response to set point and load disturbance changes is given by

$$y = \frac{PC}{1+PC} r + \frac{P}{1+PC} d \; . \tag{21}$$

For illustration purposes, assume that the plant can be modeled with first-order dynamics. Using standard IMC controller design practices as detailed in Example 1, Eq. 21 becomes

$$y = \frac{1}{\lambda s + 1} r + \frac{K_p \lambda s}{(\lambda s + 1)(\tau s + 1)} d \; , \tag{22}$$

where K_p is the process gain and τ is the process time constant. For systems where $\tau \le \lambda$, the response to changes in the set point is similar to the response to changes in the load disturbance. However, for systems where $\tau >> \lambda$, the response to changes in the set point will be quick whereas the response to the load disturbance will be limited by the process time constant. For this case, load disturbance rejection characteristics can be improved by a better selection of *Q.* In practice, this can be accomplished by judiciously selecting β in Eq. 11. For n = 2,

$$F = \frac{\beta s + 1}{(\lambda s + 1)^2} \; . \tag{23}$$

Using Eq. 23 along with relation $Q = P^{-1}F$, the IMC load disturbance rejection transfer function becomes

$$\frac{y}{d} = \frac{(\lambda^2 s + 2\lambda - \beta) s K_p}{(\lambda s + 1)^2 (\tau s + 1)} \; . \tag{24}$$

To cancel the plant dynamics in the load disturbance transfer function, set

$$\beta = 2\lambda - \frac{\lambda^2}{\tau} \tag{25}$$

subject to the constraint $\lambda < 2\tau$, which keeps the controller dynamics in the left half plane. With β properly designed, the modified IMC controller reduces to a standard PI controller of the form

$$C = \frac{\tau\beta}{\lambda^2 K_p}\left(1 + \frac{1}{\beta s}\right). \tag{26}$$

Fig. 6 illustrates the improved load disturbance rejection characteristics using Eq. 26 for a first-order system with $K_p = 1$, $\tau = 10$, and $\lambda = 3$. In this figure, the closed-loop response has been plotted for a unit change in the set point followed by a unit change in the load disturbance.

Further improvements in the load disturbance rejection characteristics – at the expense of the set point response – can be attained by decreasing λ.

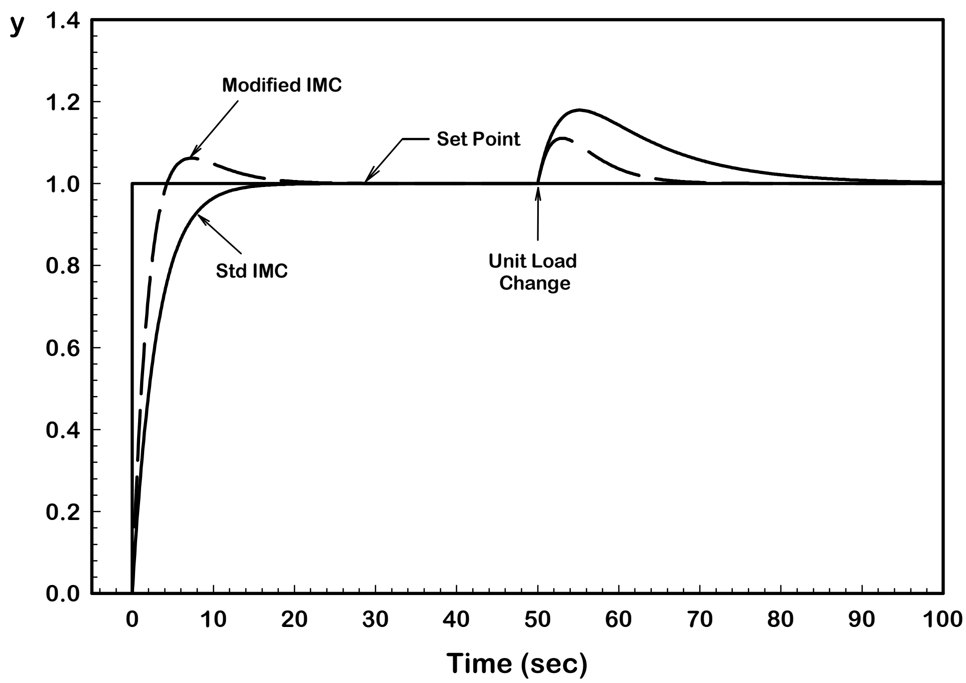

Fig. 6: Improved load disturbance rejection characteristics using Eq. 26

IMC PID tuning parameters for common process dynamics

The IMC tuning strategy for PID controllers was illustrated in Example 1 for a first-order model and in Example 2 for a first-order plus dead-time model. This procedure can be extended to other, low-order process dynamics.

In practice, process control equipment manufacturers implement PID controllers in one of three fashions. It is important to keep in mind that the IMC tuning parameters are dependent on the form of PID control implemented. Transfer functions for interacting (series), non-interacting (standard or ISA), and parallel representations of PID control are summarized in Table 1. Corresponding IMC tuning parameters are presented in Table 2 for common, low-order process models without dead times. Table 3 summarizes IMC tuning parameters for the same processes with dead time estimates. In some instances, there may be not be a unique solution, in which case, all solutions are given.

Table 1: Summary of PID controllers

PID Controller Form	Transfer Function
Interacting	$C = K\left(1+\frac{K_i}{s}\right)(1+K_d s)$
Non-Interacting	$C = K\left(1+\frac{K_i}{s}+K_d s\right)$
Parallel	$C = K+\frac{K_i}{s}+K_d s$

To be sure, these tables are not the last words in IMC. Extensions to minimum variance controllers can be found in Braatz [4], and Doyle et al. [2] generalize these ideas to include robust controller design. Nevertheless, IMC provides a general framework for systematically designing controllers and has become an important tool in the process industries.

References

[1] Skogestad, S. and I. Postlethwaite (1996). *Multivariable Feedback Control: Analysis and Design*, Wiley, New York.

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[4] Braatz, R.D., (1996). "Internal Model Control," *The Control Handbook*, W.S. Levine, ed., CRC Press.

[5] Chien, I-L., and P.S. Fruehauf (1990). "Consider IMC Tuning to Improve Controller Performance," *Chem, Engr. Prog.*, vol. 86, no. 10, pp 33-41.

Table 2: PID tuning sets for common, low-order process dynamics

Process Model	Closed-Loop Transfer Function	Interacting PID K	Interacting PID K_i	Interacting PID K_d	Non-Interacting PID K	Non-Interacting PID K_i	Non-Interacting PID K_d	Parallel PID K	Parallel PID K_i	Parallel PID K_d
$\frac{K_p}{\tau s+1}$	$\frac{1}{\lambda s+1}$	$\frac{\tau}{\lambda K_p}$	$\frac{1}{\tau}$	–	$\frac{\tau}{\lambda K_p}$	$\frac{1}{\tau}$	–	$\frac{\tau}{\lambda K_p}$	$\frac{1}{\lambda K_p}$	–
$\frac{K_p}{s}$	$\frac{1}{\lambda s+1}$	$\frac{1}{\lambda K_p}$	–	–	$\frac{1}{\lambda K_p}$	–	–	$\frac{1}{\lambda K_p}$	–	–
	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2}{\lambda K_p}$	$\frac{1}{2\lambda}$	–	$\frac{2}{\lambda K_p}$	$\frac{1}{2\lambda}$	–	$\frac{2}{\lambda K_p}$	$\frac{1}{K_p\lambda^2}$	–
$\frac{K_p}{s(\tau s+1)}$	$\frac{1}{\lambda s+1}$	$\frac{1}{\lambda K_p}$	–	τ	$\frac{1}{\lambda K_p}$	–	τ	$\frac{1}{\lambda K_p}$	–	$\frac{\tau}{\lambda K_p}$
	$\frac{2\lambda s+1}{(\lambda s+1)^2}$	$\frac{2}{\lambda K_p}$	$\frac{1}{2\lambda}$	τ	$\frac{\tau+2\lambda}{\lambda^2 K_p}$	$\frac{1}{\tau+2\lambda}$	$\frac{2\lambda\tau}{\tau+2\lambda}$	$\frac{\tau+2\lambda}{\lambda^2 K_p}$	$\frac{1}{\lambda^2 K_p}$	$\frac{2\tau}{\lambda K_p}$
		$\frac{\tau}{\lambda^2 K_p}$	$\frac{1}{\tau}$	2λ						
$\frac{K_p}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{\lambda s+1}$	$\frac{\tau_1}{\lambda K_p}$	$\frac{1}{\tau_1}$	τ_2	$\frac{\tau_1+\tau_2}{\lambda K_p}$	$\frac{1}{\tau_1+\tau_2}$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$	$\frac{\tau_1+\tau_2}{\lambda K_p}$	$\frac{1}{\lambda K_p}$	$\frac{\tau_1\tau_2}{\lambda K_p}$
		$\frac{\tau_2}{\lambda K_p}$	$\frac{1}{\tau_2}$	τ_1						
$\frac{K_p(\tau_3 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{1}{\lambda s+1}$	$\frac{\tau_1}{\lambda K_p}$	$\frac{1}{\tau_1}$	$\tau_2-\tau_3$ [a]	$\frac{\tau_1+\tau_2-\tau_3}{\lambda K_p}$	$\frac{1}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_1(\tau_2-\tau_3)}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_1+\tau_2-\tau_3}{\lambda K_p}$	$\frac{1}{\lambda K_p}$	$\frac{\tau_1(\tau_2-\tau_3)}{\lambda K_p}$
		$\frac{\tau_2}{\lambda K_p}$	$\frac{1}{\tau_2}$	$\tau_1-\tau_3$ [b]	$\frac{\tau_1+\tau_2-\tau_3}{\lambda K_p}$	$\frac{1}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_2(\tau_1-\tau_3)}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_1+\tau_2-\tau_3}{\lambda K_p}$	$\frac{1}{\lambda K_p}$	$\frac{\tau_2(\tau_1-\tau_3)}{\lambda K_p}$
$\frac{K_p}{\tau^2 s^2+2\zeta\tau s+1}$	$\frac{1}{\lambda s+1}$	Cannot be realized for $\zeta < 1$			$\frac{2\zeta\tau}{\lambda K_p}$	$\frac{1}{2\zeta\tau}$	$\frac{\tau}{2\zeta}$	$\frac{2\zeta\tau}{\lambda K_p}$	$\frac{1}{\lambda K_p}$	$\frac{\tau^2}{\lambda K_p}$
$\frac{K_p(-\tau_3 s+1)}{(\tau_1 s+1)(\tau_2 s+1)}$	$\frac{-\tau_3 s+1}{\lambda s+1}$	$\frac{\tau_1}{(\lambda+\tau_3)K_p}$	$\frac{1}{\tau_1}$	τ_2	$\frac{\tau_1+\tau_2}{(\lambda+\tau_3)K_p}$	$\frac{1}{(\tau_1+\tau_2)}$	$\frac{\tau_1\tau_2}{(\tau_1+\tau_2)}$	$\frac{\tau_1+\tau_2}{(\lambda+\tau_3)K_p}$	$\frac{1}{(\lambda+\tau_3)K_p}$	$\frac{\tau_1\tau_2}{(\lambda+\tau_3)K_p}$
		$\frac{\tau_2}{(\lambda+\tau_3)K_p}$	$\frac{1}{\tau_2}$	τ_1						

[a] Derivative gain estimated using a first-order Taylor series approximation of $(\tau_2 s+1)/(\tau_3 s+1)$, which is valid for $\tau_2 >> \tau_3$
[b] Derivative gain estimated using a first-order Taylor series approximation of $(\tau_1 s+1)/(\tau_3 s+1)$, which is valid for $\tau_1 >> \tau_3$

Table 3: PID tuning sets for common, low-order process dynamics with dead times

Process Model	Dead Time Approx.	Closed-Loop Transfer Function[a]	Interacting PID			Non-Interacting PID			Parallel PID		
			K	K_i	K_d	K	K_i	K_d	K	K_i	K_d
$\frac{K_p e^{-T_d s}}{\tau s+1}$	$1-T_d s$	$\frac{1}{\lambda s+1}$	$\frac{\tau}{(\lambda+T_d)K_p}$	$\frac{1}{\tau}$	–	$\frac{\tau}{(\lambda+T_d)K_p}$	$\frac{1}{\tau}$	–	$\frac{\tau}{(\lambda+T_d)K_p}$	$\frac{1}{(\lambda+T_d)K_p}$	–
	$\frac{1-T_d s/2}{1+T_d s/2}$	$\frac{1}{\lambda s+1}$	$\frac{\tau}{(\lambda+T_d/2)K_p}$	$\frac{1}{\tau}$	$\frac{T_d}{2}$	$\frac{\tau+T_d/2}{(\lambda+T_d/2)K_p}$	$\frac{1}{\tau+T_d/2}$	$\frac{\tau T_d/2}{\tau+T_d/2}$	$\frac{\tau+T_d/2}{(\lambda+T_d/2)K_p}$	$\frac{1}{(\lambda+T_d/2)K_p}$	$\frac{\tau T_d/2}{(\lambda+T_d/2)K_p}$
$\frac{K_p e^{-T_d s}}{s}$	$1-T_d s$	$\frac{1}{\lambda s+1}$	$\frac{1}{(\lambda+T_d)K_p}$	–	–	$\frac{1}{(\lambda+T_d)K_p}$	–	–	$\frac{1}{(\lambda+T_d)K_p}$	–	–
	$\frac{1-T_d s/2}{1+T_d s/2}$	$\frac{1}{\lambda s+1}$	$\frac{1}{(\lambda+T_d/2)K_p}$	–	$\frac{T_d}{2}$	$\frac{1}{(\lambda+T_d/2)K_p}$	–	$\frac{T_d}{2}$	$\frac{1}{(\lambda+T_d/2)K_p}$	–	$\frac{T_d/2}{(\lambda+T_d/2)K_p}$
	$1-T_d s$	$\frac{(2\lambda+T_d)s+1}{(\lambda s+1)^2}$	$\frac{2\lambda+T_d}{(\lambda+T_d)^2 K_p}$	$\frac{1}{2\lambda+T_d}$	–	$\frac{2\lambda+T_d}{(\lambda+T_d)^2 K_p}$	$\frac{1}{2\lambda+T_d}$	–	$\frac{2\lambda+T_d}{(\lambda+T_d)^2 K_p}$	$\frac{1}{(\lambda+T_d)^2 K_p}$	–
	$\frac{1-T_d s/2}{1+T_d s/2}$	$\frac{(2\lambda+T_d/2)s+1}{(\lambda s+1)^2}$	$\frac{2\lambda+T_d/2}{(\lambda+T_d/2)^2 K_p}$	$\frac{1}{(2\lambda+T_d/2}$	$\frac{T_d}{2}$	$\frac{2}{(\lambda+T_d/2)K_p}$	$\frac{1}{2(\lambda+T_d/2)}$	$\frac{T_d(2\lambda+T_d/2)}{4(\lambda+T_d/2)}$	$\frac{2}{(\lambda+T_d/2)K_p}$	$\frac{1}{(\lambda+T_d/2)^2 K_p}$	$\frac{T_d(2\lambda+T_d/2)}{2(\lambda+T_d/2)^2 K_p}$
$\frac{K_p e^{-T_d s}}{s(\tau s+1)}$	$1-T_d s$	$\frac{1}{\lambda s+1}$	$\frac{1}{(\lambda+T_d)K_p}$	–	τ	$\frac{1}{(\lambda+T_d)K_p}$	–	τ	$\frac{1}{(\lambda+T_d)K_p}$	–	$\frac{\tau}{(\lambda+T_d)K_p}$
	$1-T_d s$	$\frac{(2\lambda+T_d)s+1}{(\lambda s+1)^2}$	$\frac{2\lambda+T_d}{(\lambda+T_d)^2 K_p}$	$\frac{1}{2\lambda+T_d}$	τ	$\frac{2\lambda+T_d+\tau}{(\lambda+T_d)^2 K_p}$	$\frac{1}{2\lambda+T_d+\tau}$	$\frac{\tau(2\lambda+Td)}{2\lambda+T_d+\tau}$	$\frac{2\lambda+T_d+\tau}{(\lambda+T_d)^2 K_p}$	$\frac{1}{(\lambda+T_d)^2 K_p}$	$\frac{\tau(2\lambda+Td)}{(\lambda+Td)^2 K_p}$
			$\frac{\tau}{(\lambda+T_d)^2 K_p}$	$\frac{1}{\tau}$	$2\lambda+T_d$						
$\frac{K_p e^{-T_d s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$1-T_d s$	$\frac{1}{\lambda s+1}$	$\frac{\tau_1}{(\lambda+T_d)K_p}$	$\frac{1}{\tau_1}$	τ_2	$\frac{\tau_1+\tau_2}{(\lambda+T_d)K_p}$	$\frac{1}{\tau_1+\tau_2}$	$\frac{\tau_1\tau_2}{\tau_1+\tau_2}$	$\frac{\tau_1+\tau_2}{(\lambda+T_d)K_p}$	$\frac{1}{(\lambda+T_d)K_p}$	$\frac{\tau_1\tau_2}{(\lambda+T_d)K_p}$
			$\frac{\tau_2}{(\lambda+T_d)K_p}$	$\frac{1}{\tau_2}$	τ_1						

Table 3 (cont.)

Process Model	Dead Time Approx.	Closed-Loop Transfer Function[a]	Interacting PID			Non-Interacting PID			Parallel PID		
			K	K_i	K_d	K	K_i	K_d	K	K_i	K_d
$\frac{K_p(\tau_3 s+1)e^{-T_d s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$1-T_d s$	$\frac{1}{\lambda s+1}$	$\frac{\tau_1}{(\lambda+T_d)K_p}$	$\frac{1}{\tau_1}$	$\tau_2-\tau_3$ [b]	$\frac{\tau_1+\tau_2-\tau_3}{(\lambda+T_d)K_p}$	$\frac{1}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_1(\tau_2-\tau_3)}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_1+\tau_2-\tau_3}{(\lambda+T_d)K_p}$	$\frac{1}{(\lambda+T_d)K_p}$	$\frac{\tau_1(\tau_2-\tau_3)}{(\lambda+T_d)K_p}$
			$\frac{\tau_2}{(\lambda+T_d)K_p}$	$\frac{1}{\tau_2}$	$\tau_1-\tau_3$ [c]	$\frac{\tau_1+\tau_2-\tau_3}{(\lambda+T_d)K_p}$	$\frac{1}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_2(\tau_1-\tau_3)}{\tau_1+\tau_2-\tau_3}$	$\frac{\tau_1+\tau_2-\tau_3}{(\lambda+T_d)K_p}$	$\frac{1}{(\lambda+T_d)K_p}$	$\frac{\tau_2(\tau_1-\tau_3)}{(\lambda+T_d)K_p}$
$\frac{K_p e^{-T_d s}}{\tau^2 s^2+2\zeta\tau s+1}$	$1-T_d s$	$\frac{1}{\lambda s+1}$	Cannot be realized for $\zeta<1$			$\frac{2\zeta\tau}{(\lambda+T_d)K_p}$	$\frac{1}{2\zeta\tau}$	$\frac{\tau}{2\zeta}$	$\frac{2\zeta\tau}{(\lambda+T_d)K_p}$	$\frac{1}{(\lambda+T_d)K_p}$	$\frac{\tau^2}{(\lambda+T_d)K_p}$
$\frac{K_p(-\tau_3 s+1)e^{-T_d s}}{(\tau_1 s+1)(\tau_2 s+1)}$	$1-T_d s$	$\frac{-\tau_3 s+1}{\lambda s+1}$	$\frac{\tau_1}{\varphi K_p}$ [d]	$\frac{1}{\tau_1}$	$\tau_2+\frac{\tau_3 T_d}{\varphi}$	$\frac{\tau_1+\tau_2+\tau_3 T_d/\varphi}{\varphi K_p}$	$\frac{1}{\tau_1+\tau_2+\tau_3 T_d/\varphi}$	$\frac{\tau_1(\tau_2+\tau_3 T_d/\varphi)}{\tau_1+\tau_2+\tau_3 T_d/\varphi}$	$\frac{\tau_1+\tau_2+\tau_3 T_d/\varphi}{\varphi K_p}$	$\frac{1}{\varphi K_p}$	$\frac{\tau_1(\tau_2+\tau_3 T_d/\varphi)}{\varphi K_p}$
			$\frac{\tau_2}{\varphi K_p}$	$\frac{1}{\tau_2}$	$\tau_1+\frac{\tau_3 T_d}{\varphi}$	$\frac{\tau_1+\tau_2+\tau_3 T_d/\varphi}{\varphi K_p}$	$\frac{1}{\tau_1+\tau_2+\tau_3 T_d/\varphi}$	$\frac{\tau_2(\tau_1+\tau_3 T_d/\varphi)}{\tau_1+\tau_2+\tau_3 T_d/\varphi}$	$\frac{\tau_1+\tau_2+\tau_3 T_d/\varphi}{\varphi K_p}$	$\frac{1}{\varphi K_p}$	$\frac{\tau_2(\tau_1+\tau_3 T_d/\varphi)}{\varphi K_p}$

[a] For simplicity, zeros generated by right-half plane dead time approximations have not been included in the closed-loop transfer functions.
[b] Derivative gain estimated using a first-order Taylor series approximation of $(\tau_2 s+1)/(\tau_3 s+1)$, which is valid for $\tau_2 >> \tau_3$
[c] Derivative gain estimated using a first-order Taylor series approximation of $(\tau_1 s+1)/(\tau_3 s+1)$, which is valid for $\tau_1 >> \tau_3$
[d] $\varphi = \lambda + \tau_3 + T_d$

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