The Use of Control Valve Sizing Equations with Simulation Based Process Data

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Abstract

Modern process simulations are increasingly sophisticated and account for many real fluid behaviors. This yields a plethora of data, some of which is used to specify the final control element. The data that is selected for such use must be consistent with the flow model and assumptions that underlie the governing sizing equations. Use of inappropriate data can result in sizing errors that are potentially significant or consequential.

Of particular interest is the compressible flow thermodynamic behavior embodied in the current ISA control valve sizing standards equations. This behavior is modeled as an ideal gas and characterized through the ideal gas ratio of specific heats of the flowing fluid. Attempts to account for actual fluid behavior includes the use of simulation produced ratio of specific heat values that have been adjusted for actual operating pressure and temperature. Use of data more representative of the actual value is only a partial correction for non-ideal behavior and can introduce unexpected results. Recent revisions to standards recognize this disparity and provide some recommended limits on use.

This paper examines the underlying control valve flow model and assesses the impact of using the various corrected values of the ratio of specific heats in the equations. Current standards guidelines are also discussed.

Introduction

"Valve sizing" is the process of determining whether a selected control valve has sufficient flow capacity and operating characteristics to provide the intended process control. This is a critical step to ensuring that the process functions reliably and within the desired operating and economic parameters.

Unfortunately, there is no universal "single-sided" conservatism for valve sizing. Typically most instrument engineers desire to avoid undersized valves since this would result in lowered production and revenue stream to the plant owner. However, it also has the potential to compromise a safety relief system that may be dependent on that particular valve. Likewise, an oversized valve comes with penalties as well. A valve that is oversized can result in poor process control, especially in limiting the ability to use automatic mode and higher order control functionality; it incurs excessive capital expenditure (amplified through a larger actuator and associated costs); potentially compromises safety relief systems; and results in increased startup and ongoing maintenance cost due to valve wear.

A recent market study estimated that somewhere between 5 to 15% of all control valves are incorrectly sized on initial application. This represents a purchase price of 255 to 765 million dollars on an annual, worldwide basis.

Primary sources of common valve sizing errors and discrepancies are shown in Table 1. This paper will focus on a specific aspect of the second category dealing with fluid behavior.

Table 1 – Sources of Valve Sizing Errors

Source of Error	Magnitude
1. Accuracy of full scale tested performance parameters	< 5%
2. Conformance of fluid behavior to model	1% - 30+%
3. Conformance of installed geometry to model.	1% - 50+%
4. Difference between actual and design data.	??
5. Variation between production units.	< 5%

Industry Standards

Voluntary industry standards, specifically ANSI/ISA S75.01 [1][†] and IEC 60534-2-1 [2] and their respective testing standards provide equations for predicting the flow rate of liquids and gases through a control valve. These equations are used to specify a valve size to a given process. Certain fluid behaviors and assumptions, typically idealizing, are implicit in these equations although not always obvious or stated. Process design data used to perform initial control valve sizing is often generated by sophisticated process modeling that is also predicated on specific fluid behaviors and assumptions often more representative of real fluid behavior. Problems can arise when the domains of these two fluid models are not congruous.

The following discussion specifically examines the nature of the compressible flow sizing equations as given in the standard in the context of its development domain, current domain of use and the consequences of using the model outside of intended limits.

[†] Numbers in brackets refer to citations listed in the Bibliography section.

Basis of Valve Sizing Equations

The full set of control valve sizing equations given for compressible fluids is summarized in Appendix A (an in-depth technical treatise on all aspects of this model is outside the scope of the present work). The focus herein will be restricted to those key elements of the model that have a bearing on use of real process data (or data from a more sophisticated analysis) in these equations. Three elements are particularly important:

- The implicit assumption of ideal gas behavior.
- Accounting for the density change in the gas as it flows from the inlet of the valve to the throat of the valve.
- The fact that the equations are normalized to air.

To understand the assumptions and limitations associated with this valve sizing equation, it is helpful to understand the evolution of the flow model and associated equations. In its most basic form, the flow equation relates the volumetric flow rate to the observable pressure drop across the device, the density of the fluid and an empirically determined flow coefficient. Liquids are generally considered incompressible, so the density remains constant as the fluid moves through the valve. The most basic form of the equation is therefore for liquid flows and is given by:

$$Q = C_v \sqrt{\frac{\Delta P}{G_L}} \; ^{\ddagger} \qquad (1)$$

Adaptation of this equation for use with compressible flows requires three adjustments that account for the change in density of the fluid with pressure and temperature. First, the flow rate is typically expressed in units of "standard" or "normal" volumetric flow to avoid the added complication of fluid speed in the pipe changing with line pressure. Second, although the inlet density can be supplied directly to the equation as in liquid flows (see equation A.1), it can also be incorporated into the flow equation via an established relationship between pressure, density and temperature. The ideal gas "equation-of-state" (EOS) is leveraged for both of these adjustments:

$$Pv = \frac{R}{M} T \qquad (2)$$

Third, the flow rate through a valve is governed by the fluid speed and density at the throat of the valve. The latter quantity remains constant for ideal liquid flow, but is a function of pressure for gas flows and hence changes due to the exchange between kinetic energy and static pressure. It is therefore necessary to account for the change in fluid properties at the throat of the valve.

Circa 1969 Driskell [3] introduced an "expansion factor", *Y*, that corrects the apparent flow rate through the valve for the change in gas density at the throat of the valve. His formulation followed the general academic analysis of flow nozzles and orifice plates. Theoretical considerations and laboratory testing justified two important characteristics of this parameter: a) that a linear correction for this

‡ All terms are defined in the Nomenclature section at the end of this paper.

effect as a function of pressure drop ratio was acceptable, [3,4,5] and b) a limiting value of 2/3 at fully choked flow results for all valve styles from this model choice. [3].

The limiting, or terminal, pressure drop at which choked flow occurs depends on valve geometry and must be determined empirically for each valve style. This condition is expressed in the form of the pressure drop ratio, x, in a term known as the critical pressure drop ratio factor, x_T. The resulting expression for the expansion factor, Y, was:

$$Y = 1 - \frac{x}{3x_T} \tag{3}$$

In this equation and the general flow equation, the pressure drop ratio, x, is limited to a maximum value of x_T.

Incorporating the expansion factor into the basic liquid flow equation yields the general compressible flow model:

$$Q = C_v Y \sqrt{\frac{\Delta P}{G_L}} \tag{4}$$

This proved to be an acceptable model for compressible fluid flow through valves. However, in this form the correction for fluid expansion is unique to the test fluid so use of the model would be restricted to that single fluid. To avoid this impracticality an adjustment factor to the x_T values is required.

Correcting the expansion process to the throat requires an understanding of the nature of that expansion and the factors affecting it. The expansion of the gas from the inlet to the throat of a single stage throttling device is generally considered to be relatively reversible and adiabatic so that entropy remains constant. Thermodynamics provide a relationship between pressure and specific volume (inverse of density) for ideal gas isentropic flows:

$$Pv^{\gamma} = \text{constant} \tag{5}$$

This establishes a dependency of the density change in expanding to the throat on the ratio of specific heats.

Again following the schema employed for flow nozzles and orifice plates, analysis and testing of air and steam testing over the range $1.08 < \gamma < 1.65$ yielded the following linear correction to the terminal pressure drop ratio:

$$F_{\gamma} = \frac{\gamma}{\gamma_{air}} = \frac{\gamma}{1.4} \tag{6}$$

The resulting final expansion factor model is thus:

$$Y = 1 - \frac{x}{3F_\gamma x_T} \tag{7}$$

Accordingly, the pressure drop ratio used in the both equations (4) and (7) is limited to the value of $F_\gamma x_T$.

This fundamental model in conjunction with measured flow coefficients has provided flow rate prediction suitably accurate for valve sizing and selection for more than four decades when used within the following limitations:

1. Ideal gas behavior
2. $C_v/d^2 \leq 30$
3. $1.08 < \gamma < 1.65$

Current Industrial Context

The capability to simulate real process behaviors is increasingly prevalent. Values for fluid properties and thermodynamic processes routinely represent real behavior that goes beyond the ideal gas behavior assumptions implicit in the valve sizing equations. The challenge facing the present day practitioner is whether the data available to them can be used appropriately within existing flow models (sizing equations). Specifically, can the corrections for different gases be used to also correct for non-ideal gas behavior?

Gases are generally considered to behave in an ideal fashion when the molecular spacing is large in comparison to the gas molecule itself. Under this condition, the intermolecular forces are generally negligible and molecular interaction is largely kinetic. Two characteristics of ideal gas behavior are a) they conform to the ideal gas EOS presented earlier (equation (5)), and b) that the ratio of specific heats remains constant (a perfect gas) or changes only with temperature (an ideal gas) [6].

Molecular spacing of the gas molecules is compressed at relatively high pressures and temperatures such that molecular forces become a factor. Properties such as the ratio of specific heats are no longer constant, but become strong functions of temperature and pressure. Non-ideal (i.e., "real") gas flow behavior can depart significantly from models based on simple ideal gas behavior. This must be taken account in the context of valve sizing and consideration given to the impact on both the *PvT* and isentropic expansion behaviors integral to the gas flow model.

The former is readily addressed by either supplying actual density to the sizing equation (e.g., equation A.1) or utilizing a modified gas EOS to represent the density in the equations. The following equation is often used for this purpose:

$$Pv = \frac{ZRT}{M} \tag{8}$$

This modified form of the ideal gas equation of state utilizes the compressibility factor, *Z*, to restore the pressure, temperature and specific volume to a correct relationship. This factor may be evaluated from various higher order equations of state (e.g., cubic EOS), generalized compressibility factor charts based on other equations of state, or from actual empirical data. Sizing equations A.2 and A.3 are based on this approach.

The polytropic expansion of the gas from the inlet to the throat of the valve, as previously noted, is generally assumed to be isentropic. Under non-ideal conditions the equation that characterizes this expansion is:

$$Pv^{n_v} = \text{constant} \tag{9}$$

Similar in appearance to equation (5), difference lies in the fact that the exponent is a function of temperature and pressure, and is not numerically equal to the ratio of specific heats. The exponent, n_v, is known as the "isentropic volume exponent" and is given by:

$$n_v = -\frac{v}{P}\left(\frac{\partial P}{\partial v}\right)_T \frac{c_p(P,T)}{c_v(P,T)} \tag{10}$$

Only under the ideal or perfect gas assumption does this exponent reduce to the ratio of specific heats. Typical values of γ for many gases fall near a value of 1.4 and the effect of the linear F_γ and any associated inaccuracies is slight. However, at elevated temperatures and pressures where non-ideal behaviors exist the value of the isentropic exponent can be significantly different (typically greater) than the ideal gas ratio of specific heats, e.g., on the order of two to three times larger. Such large values fall outside the range of values over which the sizing equation was originally developed and it raises the question of whether the flow model is suitable for use under such circumstances.

The effect of utilizing the ideal gas model in the non-ideal gas region can be assessed by analytically investigating the behavior of flow through a nozzle under both ideal and non-ideal conditions. A set of nozzle flow equations analogous to the valve sizing equation are presented in appendix B (these equations are the basis for much of the seminal work previously presented on valve sizing equation development). Whereas the independent pressure differential in the nozzle equations is from the inlet to the throat (or vena contracta), the pressure drop ratio terms, x and x_T are adapted to throat conditions instead of downstream conditions and denoted by x_{vc} and x_{vcT}.

Equation B.1 will be employed to represent the expected real flow rate through the nozzle. This equation is applicable over the full range of compressible conditions including non-ideal gas conditions as long as the average isentropic exponent is used. This holds since the equation derives form fundamental equations which are valid over real gas conditions.

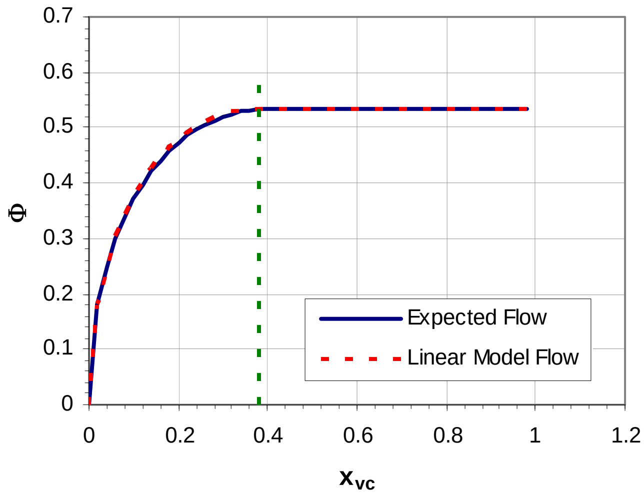


Figure 1 – *Ideal gas flow; ideal gas behavior flow model (γ = 1.14).*

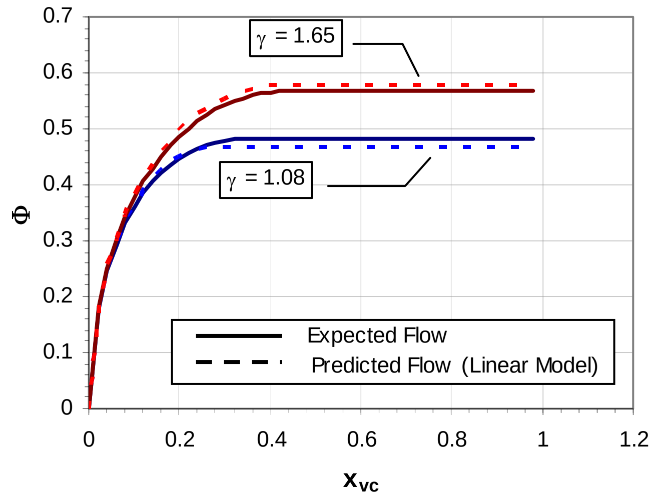


Figure 2 – *Case A: Ideal gas flow at γ = 1.08; γ = 1.65; ideal gas behavior flow model.*

Ideal vs. Non-Ideal Comparative Analysis

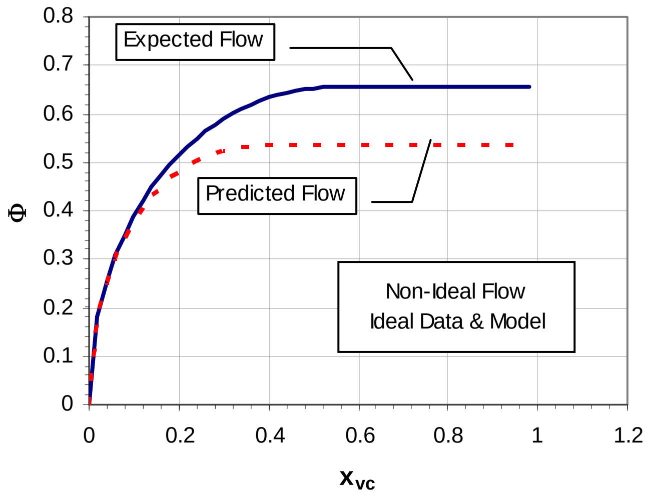
Baseline data is generated for air at ambient conditions (γ = 1.4). Equation (B.1) is used to generate a flow rate vs. vena contracta pressure drop ratio, x_{vc}. This curve represents expected actual flow and is used to determine the value of x_{vcT} (0.379)for this device per equation (B.8).

This data is then used in the linear ideal flow model, Equation (B.3), to generate a comparative predicted curve. The resulting flow curves are shown in Figure 1. As expected, good agreement between the simulated actual flow and the flow predicted by the linear flow model is obtained.

Four combinations of actual fluid behavior, data and sizing equation model are examined:

Case	Process Fluid Behavior	Process Fluid Data	Sizing Model
A	Ideal	Ideal	Ideal
B	Non-Ideal	Ideal	Ideal
C	Non-Ideal	Non-Ideal	Ideal
D	Non-Ideal	Non-Ideal	Non-Ideal

Case A: Similar comparisons between expected fluid behavior and predicted flow behavior were conducted in the ideal gas range at the limits of specific heat ratio over which the equation was developed (γ = 1.08; γ = 1.65). These results are presented in Figure 2, and again show good agreement with predicted flow rate, falling within ±3% of the simulated actual flow rate.



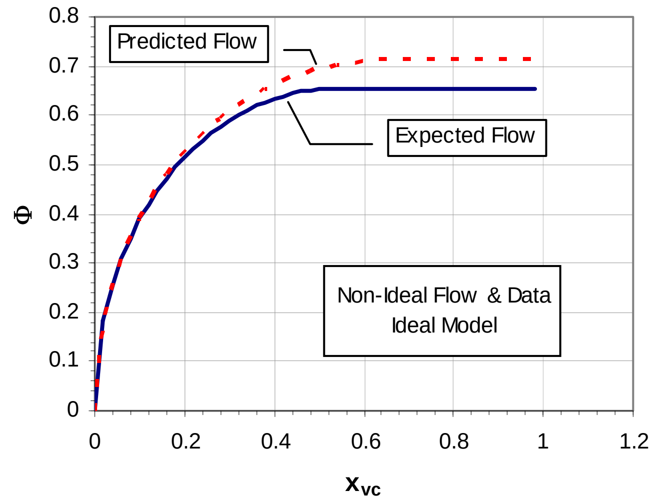


Figure 3 – *Case B: Non-ideal gas flow (n_v=2.5); ideal gas behavior flow model using ideal gas ratio of specific heats (k=1.3)..*

Figure 4 – *Case C: Non-ideal gas flow (n_v=2.5); ideal gas behavior flow model using isentropic. exponent to evaluate ratio of specific heats (k=2.5)*

Case B: Use of the ideal gas behavior flow model and data to represent fluid behavior in the non-ideal gas range results in significant disparity between predicted flow and expected real flow rate. Figure 3 presents results for a hypothetical gas with γ=1.3, but operating at a pressure and temperature where the average isentropic exponent over the expansion from inlet to throat is 2.5. In this case, the isentropic expansion is not characterized well by the ratio of specific heats and the ideal gas model, and under predicts flow rate in excess of 18%. This would result in an oversized valve.

Case C: Using a value of the isentropic exponent that better characterizes the real expansion in the ideal model still results in unacceptable coherence between the predicted and simulated actual flow rates. In this case the average isentropic exponent of 2.5 is used instead of the ideal gas ratio of specific heats in the ideal flow model. The disparity is again large (> 9%), but in this case the flow rate is over predicted, resulting in an undersized valve. These results are shown in Figure 4.

Case D: The subject of non-ideal gas flow through control valves has been previously studied by Fagerlund [7] and Riveland [8]. These studies revealed the non-ideal effects to be pronounced and that while the linear nature of the expansion factor as a function of pressure drop ratio appears justified, the linear nature of F_γ (equation 6) breaks down. A modified non-linear expression developed under assumptions comparable to the original ideal gas development was proposed as:

$$F_n = 1.223\sqrt{n_v} - 0.443 \tag{11}$$

This correction factor is analogous to F_γ and used in the same manner to adjust the terminal pressure drop (pressure drop at which choked flow occurs).

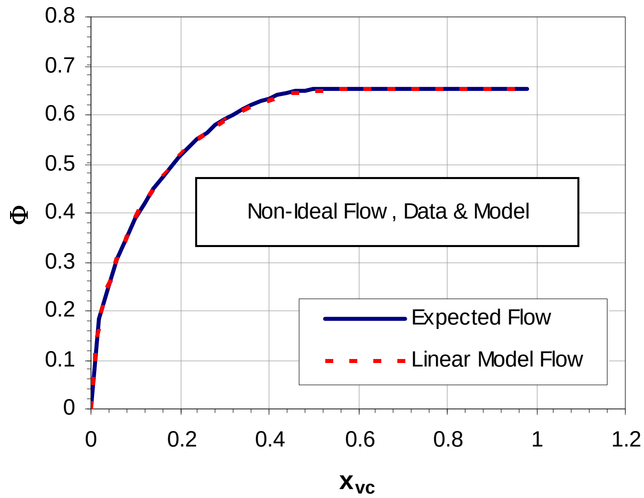


Figure 5 – *Case D: Non-ideal gas flow (n_v=2.5); non-ideal behavior flow model using isentropic exponent .ratio of specific heats (k=1.3)..*

$$x_{choked} = F_n x_T \tag{12}$$

Repeating the analysis of Case C employing this correction factor results in superior agreement between the simulated actual flow and predicted flow curves as shown in Figure 5 (± 0.5%). It can also be shown that this is valid under ideal gas conditions and renders good agreement to the F_γ model in that range.

Conclusions

The linear nature of both the expansion factor, *Y*, and the specific heat ratio factor, F_γ, in the industry standard sizing equations is justified for ideal gas behavior. Flow rates are adequately modeled over pressure drop ratio ranges up to the point of choked flow for fluids with specific heat ratios that fall in the range of $1.08 < \gamma < 1.65$.

The linear nature of F_γ is not adequate under non-ideal gas behavior, and a more appropriate gas behavior model should be employed in sizing. The consequences of using an ideal gas behavior flow model to size valves in the non-ideal gas range are varied depending on the combination of fluid behavior, fluid behavior model and data. These are summarized in Table 2.

Table 2 – Summary of Sizing Model and Data Combinations.

Process	Data	Model	Comments
Ideal	Ideal	Ideal	Acceptable to use ANSI/ISA S75.01.01 equations. The forthcoming revision of the standard advises that use of the equations is acceptable when the ratio of specific heats falls within the range 1.08 < γ < 1.65. This range corresponds to the range over which the model was originally developed and was intended to correct for different gases.
Real	Ideal	Ideal	Depending on the degree of departure from ideality, errors in excess of 10% may result from using the ideal model to simulate real behavior. In general, the ideal model is likely to under predict the expansion factor, Y, and therefore overall through put. If the real isentropic exponent falls within the above prescribed range, it is expected that the accuracy of the equations as stated in the standard is preserved.
Real	Real	Ideal	Depending on the degree of departure from ideality, errors in excess of 10% may result from using the real isentropic exponent in the ideal gas model. In general, this scenario will over predict the expansion factor, Y, and therefore over predict the overall throughput. If the real isentropic exponent falls within the above prescribed range, it is expected that the accuracy of the equations as stated in the standard is preserved.
Real	Real	Real	No official "real" model has been adopted by the standards at this point in time. An adaptation of the standard control valve flow equations for real gas conditions has been proposed. This adaptation is theoretical in nature and follows logic similar to the original development of the ideal gas equations. The accuracy has not been rigorously quantified, but is expected to be on the order of +/- 10% or less.

Recommendations

For maximum accuracy it is recommended that the valve sizing equations presented in [1] and [2] be used within the limits stated in the standard with special emphasis on the valid range of specific heat ratio.

For applications falling outside the range of specific heat ratio associated with ideal gas behavior, it is recommended that a non-ideal gas behavior model such as that proposed in [8] be utilized along with the average isentropic exponent over the expansion from inlet to throat.

Nomenclature

Symbol	Definition
A_{vc}	Cross sectional flow area at vena contracta.
C_v	Valve sizing equation flow coefficient.
c_p	Specific heat capacity at constant pressure.
c_v	Specific heat capacity at constant volume.
d	Nominal valve size, in.
F_n	Isentropic exponent factor for non-ideal gas.
F_F	Specific heat ratio factor.
G_L	Liquid specific gravity.
M	Molecular mass of flowing fluid.
n_v	Isentropic volume exponent
P	Mean static fluid pressure.
Q	Volumetric flow rate.
r	Ratio of local pressure to upstream pressure (P/P_1).
R	Universal gas constant.
T	Absolute fluid temperature.
v	Specific volume.
W	Mass flow rate.
x	Pressure drop ratio ($\Delta P/P_1$).
x_{choked}	Choked pressure drop ratio for compressible flow.
x_{sizing}	Value of pressure drop ratio used in computing flow for compressible fluids.
x_T	Pressure drop ratio factor of a control valve.
x_{vc}	Pressure drop ratio to vena contracta ($\Delta P_{vc}/P_1$).
x_{vcT}	Pressure differential ratio factor for a nozzle based on vena contract pressure differential.
Y	Expansion factor.
ΔP	Pressure differential applied across a throttling device.
ΔP_{vc}	Pressure differential from inlet to vena contracta of throttling device.
ν	Fluid specific volume.
Z	Gas compressibility factor
ρ	Fluid density.
β	Ratio of vena contracta diameter to inlet diameter.
Φ	Mass flux.

Subscripts

Symbol	Definition
1	Conditions upstream of throttling device.
2	Conditions downstream of throttling device.
vc	Vena contracta.

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Appendix A
Compressible Flow Control Valve Sizing Equations

The following equations constitute the fundamental compressible flow sizing model presented in the ISA and IEC standards. Piping correction terms (F_P, x_{TP}) have been omitted for clarity.

$$W = C_v N_6 Y \sqrt{x_{sizing} P_1 \rho_1} \tag{A.1}$$

$$W = C_v N_8 P_1 Y \sqrt{\frac{x_{sizing} M}{T_1 Z_1}} \tag{A.2}$$

$$Q_s = C_v N_9 P_1 Y \sqrt{\frac{x_{sizing}}{M T_1 Z_1}} \tag{A.3}$$

These are equivalent equations based on the same flow model. Equation (A.1) is the basic sizing equation. Equation (A.2) is derived by substituting the fluid density as computed from the ideal gas equation-of-state into equation (A.1). Equation (A.3) expresses the flow rate in standard volumetric units.

$$x_{sizing} = \begin{cases} x & if \;\; x < x_{choked} \\ x_{choked} & if \;\; x \geq x_{choked} \end{cases} \tag{A.4}$$

Where,

$$x = \frac{\Delta P}{P_1} \tag{A.5}$$

$$x_{choked} = F_\gamma x_T \tag{A.6}$$

$$F_\gamma = \frac{\gamma}{1.4} \tag{A.7}$$

$$Y = 1 - \frac{x_{sizing}}{3 x_{choked}} \tag{A.8}$$

Constant		**Formulae unit**				
	C_v	**W**	**Q**	**P, ΔP**	**ρ**	**T**
N_6	2.73	kg/h	–	kPa	kg/m³	–
	2.73×10^1	kg/h	–	bar	kg/m³	–
	6.33×10^1	lbm/h		psia	lbm/ft³	
N_8	9.48×10^{-1}	kg/h	–	kPa	–	K
	9.48×10^1	kg/h	–	bar	–	K
	1.93×10^1	lbm/h		psia		°R

Constant		Formulae unit				
	C_v	*W*	*Q*	*P*, ΔP	ρ	*T*
N_9 (t_s = 0 °C)	2.12×10^1 2.12×10^3 6.94×10^3	– –	m³/h m³/h scfh	kPa bar psia	– –	K K °R
N_9 (t_s = 15 °C)	2.25×10^1 2.25×10^3 7.32×10^3	– –	m³/h m³/h scfh	kPa bar psia	– –	K K °R

Appendix B
Compressible Flow Nozzle Flow Equations

The following is a generalized flow equation associated with a nozzle. This equation derives from fundamental equation valid over non-ideal gas conditions. It is applicable over the full range of compressible conditions as long as the average isentropic exponent is used in lieu of the ratio of specific heats.

$$\Phi = \frac{w}{A_{vc}\sqrt{2P_1\rho_1}} = \sqrt{\left(\frac{n_v}{n_v - 1}\right)\frac{r^{2/n_v}\left(1 - r^{\frac{n_v - 1}{n_v}}\right)}{1 - \beta^4 r^{2/n_v}}} \tag{B.1}$$

$$r = \frac{P_{vc}}{P_1} \tag{B.2}$$

The following is the equivalent nozzle flow equation corresponding to the control valve flow model.

$$\Phi = Y\sqrt{\frac{x_{vc}}{1 - \beta^4}} \tag{B.3}$$

$$x_{vc} = \frac{P_1 - P_{vc}}{P_1} \tag{B.4}$$

$$x_{vc} = 1 - r \tag{B.5}$$

$$x_{sizing} = \begin{cases} x & if\ \ x < x_{choked} \\ x_{choked} & if\ \ x \geq x_{choked} \end{cases} \tag{B.6}$$

$$Y = 1 - \frac{x_{vc}}{3x_{vcT}} \tag{B.7}$$

$$x_{vcT} = \left(\frac{3\Phi_{choked}}{2}\right)^2\left(1 - \beta^4\right) \tag{B.8}$$