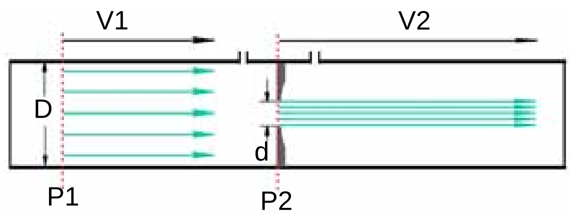
DP Flow Theory - Deriving the DP Flow Equation (Part 1)



The theoretical DP flow equation is derived from two sources: Bernoulli's equation and the continuity Equation. Bernoulli's equation states that the sum of the pressure energy (*P*), kinetic energy ($\frac{1}{2}\rho v^2$) and potential energy $(\rho g h)$ in a pipe will be equal at any cross section.

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \rho v_2{}^2 + \rho g h_2$$

The continuity equation states that the mass flow through any cross section in a pipe will be equal. By assuming density is constant, the continuity equation can be written: $A_1 V_1 = A_2 V_2$ We'll abbreviate all of the algebra steps as follows:

1. Cancel the potential energy terms in Bernoulli's Equation (assume the pipe is level)
2. Solve for v_1 in terms of v_2 using the continuity equation
3. Substitute v_1 in terms of v_2 in Bernoulli's equation and simplify
4. Solve for the velocity at the throat, v_2

$$v_2 = \sqrt{\frac{2\Delta P}{\rho\left[1 - \frac{d^4}{D^4}\right]}}$$

This is the equation for the velocity at the meter throat. Since most people don't measure their flow rates in velocity, the volumetric flow (Q_v) equation can be obtained by multiplying by the cross sectional area of the throat.

$$Q_v = \frac{\pi}{4}d^2\sqrt{\frac{2\Delta P}{\rho\left[1 - \left(\frac{d}{D}\right)^4\right]}}$$

If mass flow (Q_m) is required rather than volumetric flow, multiply both sides by the flowing density.

$$Q_m = \frac{\pi}{4}d^2\sqrt{\frac{2\Delta P\rho}{1 - \left(\frac{d}{D}\right)^4}}$$

To understand DP flow thoroughly, it is important to understand the theory. We all know that flow rate is proportional to the square root of DP, but what does the DP really measure? Most other flowmeters measure something that is directly proportional to velocity or directly proportional to the mass flow rate. Vortex meters and coriolis are good examples. But what is DP directly proportional to?

DP actually measures the kinetic energy, the $\frac{1}{2}\rho v^2$ term, in the equation. If the kinetic energy and the density is known, it is possible to solve for velocity.

$$v \;\alpha\; \sqrt{\frac{2\Delta P}{\rho}}$$

This relationship has two inherent advantages. In an un-compensated mass or standard volume measurement, a DP flowmeter will be 1/2 as sensitive to changes in fluid density as a velocity type flowmeter. Second, errors in measured DP will only contribute 1/2 as much error to the measured flow rate. So a 0.5% error in DP measurement will only cause a 0.25% error in measured flow.

DP Flow Theory Part II will discuss the empirical corrections to the theoretical equation above.

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