technical monograph 13A

Frequency Response for Process Control Elements

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INTRODUCTION

An increasing concern for the dynamic performance of industrial process control systems has stimulated interest in the frequency response of individual process control elements. This interest is usually justified by the concise picture of over-all dynamic behavior that experimental frequency response can provide. It has also aroused many questions regarding the significance of the effects of load conditions and component adjustments as well as the relationship between laboratory and field conditions.

Part I of this manual is a general discussion of the procedures and practices used in the acquisition of frequency response data. The influence of various operating conditions on the validity of laboratory test data is also discussed. Part II is devoted to various mathematical concepts. Both linear and non-linear functions are discussed and equations are presented for some of the basic control elements.

PART I

FREQUENCY RESPONSE DEFINED

The dynamics of any device can be experimentally measured by comparing some input disturbance with the resulting output signal change. The type of input disturbance or test signal used depends upon the nature of the equipment to be studied, the test facilities available and the philosophy of the experimenter. Commonly used test signals include the impulse, step, ramp and sinusoid. The frequency response method utilizes a sinusoidal test signal of constant amplitude at various frequencies. Strictly speaking, frequency response has significance only for linear elements, elements whose behavior can be described by linear mathematical equations. Actually the appearance of certain common non-linearities does very little to reduce the usefulness of the technique. In general, any device suitable for frequency response analysis, when subjected to a sinusoidal input will produce a sinusoidal output of the same frequency. In addition, if the test signal amplitude is held constant but if the frequency is varied, two effects may be observed in the output sinusoid. The output amplitude will vary with frequency, generally decreasing at higher frequencies and the output wave will be shifted in phase with reference to the input wave, generally with the output lagging at higher frequencies.

Frequency response analysis can be summarily described as subjecting a device to a constant amplitude sinusoidal disturbance and observing the output amplitude and phase shift as functions of the test signal frequency.

Test Method and Equipment

It is apparent from the preceding section that two items of equipment are required to perform a frequency response test - a test signal generator which is capable of providing a suitable sine wave input, and a recorder capable of measuring both input and output signals. Normally, an electronic sine wave generator is used, and when supplemented with the appropriate transducers, it can provide a test signal of any required quantity such as force, pressure, motion, voltage or current. The recording device is usually a two-channel strip chart recorder with sensing and transducing devices permitting it to record any of these quantities. Fig. 1 illustrates the basic test arrangement. Certain conveniences can be realized with other equipment combinations, though the principles of operation are perhaps less obvious.

Reduction of Test Data

Two terms which must be fully understood before we can proceed with a discussion of frequency response

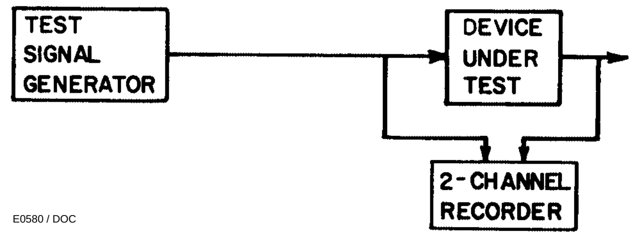

Figure 1. Schematic of Frequency Response Test Arrangement

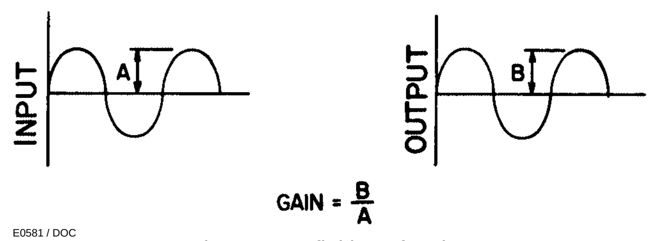

Figure 2. Definition of Gain

are *gain, normalized gain* and *phase shift.* Gain is the ratio of output change to input change. When the input to a device is a sinusoidal signal of constant amplitude and frequency, as shown in Fig. 2, the output will be a sinusoid of constant amplitude, the same frequency, but may differ from the input in both amplitude and units. The ratio of output amplitude to input amplitude is called the gain of the device.

If, however, a step change is made in the input to a device and sufficient time is permitted to allow all transients to die out before measuring the output change, the resulting ratio is called the static or zero frequency gain. Fig. 3 illustrates this quantity.

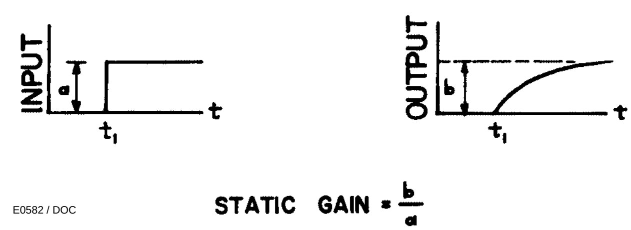

Figure 3. Definition of Static Gain

At sufficiently low frequencies the gain and static gain of a device will be equal. As the test signal frequency is increased, the gain of the device will differ from the static gain and may either increase or decrease. The ratio of the actual gain at any frequency to the static gain is called the *normalized gain* for that frequency. A plot of normalized gain vs frequency would thus indicate how the gain of the device varies from the static gain as a function of frequency.

Phase shift is a measure of the apparent delay or anticipation of the output of the device in response to an input signal. When the input signal is a sinusoid, one full cycle is considered to represent 360°. If the output sinusoid is occurring simultaneously with the input signal, no phase shift exists. The output sinusoid may lag or lead the input signal by some portion or multiple of a cycle. The phase shift is measured as that portion or multiple of a cycle multiplied by 360°. The phase shift will usually be a function of frequency and can be plotted as such. Fig. 4 illustrates typical phase measurements at two different frequencies.

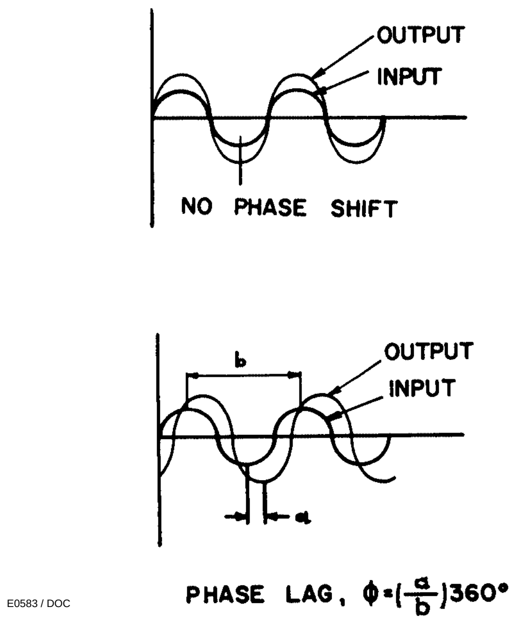

Figure 4. Measurement of Phase Shift

Presentation of Data

Although there are a number of methods of plotting frequency response data, the Bode (pronounced Bōdē) plot is probably by far the most popular. Two curves are used with this method. The phase shift and logarithm of the normalized gain are each plotted against the logarithm of the test signal frequency. A typical Bode plot is shown in Fig. 5. The scale on the gain curve illustrates an important option in presenting this data. The gain may either be displayed on logarithmic coordinates or the logarithm of the gain may be displayed on linear coordinates. In either case the shape of the curve will be the same. Certain conveniences arise in the application of this data, however, if the latter option is used. The most common method of plotting the log of the gain is to use the relationship,

$$db = 20 log_{10} M$$

where:

db is the normalized gain in decibels
M is gain

In addition to presenting the test data, a description of the device tested and the loading conditions must be recorded. Figure 6 shows a typical Bode chart with the necessary descriptive material.

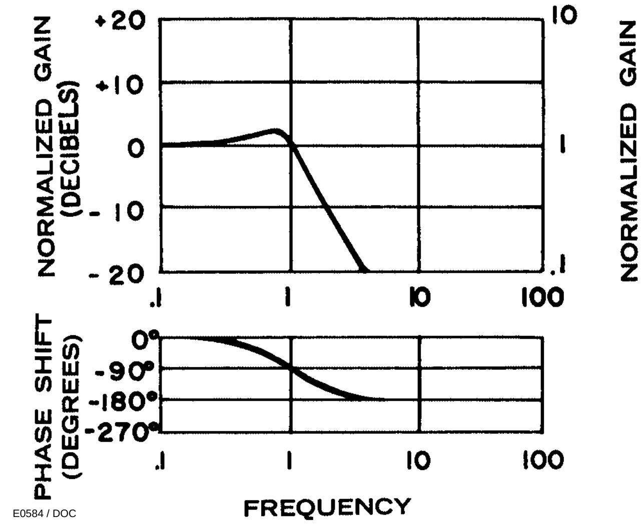

Figure 5. Typical Bode Plot

The title block on the right begins with the name and type number of the device tested and the test date. Below this are two sections describing the output and input conditions. The “Output Range” is the normal full-scale excursion of the output as identified by the end-scale values. In those devices where the range is adjustable, the response may be a function of the adjustment. “Output Mean Level” gives the mean output value about which the test was performed and is given as a per cent of the full-scale output span. Usually the mean level is at mid-range or 50%. “Output Location” indicates where the output of the device was measured for the test being reported. “Output Load” is used to describe the loading conditions imposed upon the device during the test.

The “Input Range” is the normal full-scale excursion of the input variable as identified by the end-scale values. “Input Amplitude” gives, as a per cent of the full-scale value, the amplitude at which the test was performed. Normally this will be no larger than a few per cent. The “Input Location” indicates where the input was measured during the test.

Below the input and output sections, the “Static Gain” is recorded. This may be calculated from the input and output ranges and is listed merely as a convenience. The next item, labeled “Supply”, describes the media and level of the power supply. Further down is the laboratory reference code and any special remarks that may help clarify the device under test or the test conditions.

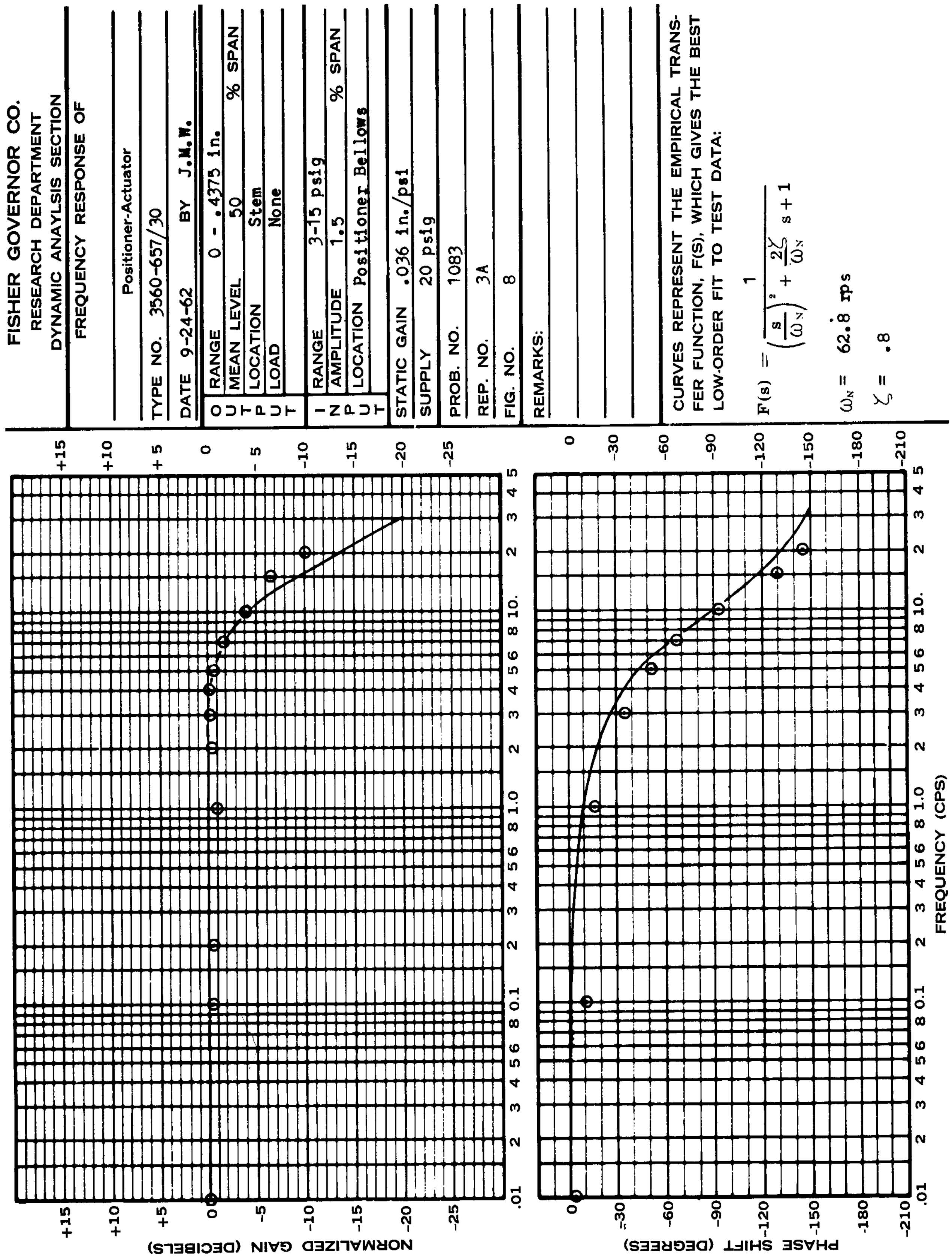

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Figure 6. Typical Frequency Response Plot

The title block on the frequency response paper and the data plotted with it completely describe the device under test, the test conditions and the dynamic performance of the device. Often it is desired to have the performance information presented in the form of an equation rather than graphically. The transfer function, which will be discussed further in Part II, is usually the most desired equation form. Fig. 6 illustrates a typical frequency response plot. The empirical transfer function which gives the best low-order fit to the test data is given in the lower section of the title block. The curves drawn through the test data represent the transfer function so that the closeness of fit between the test data and the equation can easily be observed.

Influence of Test Conditions

There are many test conditions which have an influence on the frequency response of a control device. There are also some differences between field service and laboratory conditions. The problem of establishing the most appropriate test conditions to assure that the experimental data will be valid during field operation will now be considered.

Amplitude

Among the initial problems in establishing test conditions is that of determining an appropriate test signal amplitude. Almost all control devices are subject to two amplitude dependent non-linear effects; dead band and saturation.

Dead band is the smallest input signal change to which a device can respond and is usually the result of static friction or the basic threshold of some element in the device. If the test amplitude is below the dead band value, the gain of the device is zero. Once the amplitude is appreciably greater than the dead band, however, its existence has very little effect.

Saturation occurs when the test amplitude is so large that some element reaches the limit of its output. Examples of saturation are travel stops which limit the motion of a mechanical part or a valve which may exceed the travel for which it can effectively produce a flow change. The effect of saturation is to reduce substantially the gain at higher frequencies, that is, reduce the band width. All test amplitudes between these extremes will give virtually identical results on devices which are suitable for frequency response analysis. The acceptable amplitude range depends upon the type of equipment but typically could extend from 0.2% up to 5%. The range of amplitude between dead band and saturation is the only amplitude range for which frequency response has any significance. The usefulness of this data in a stability analysis is still very great. If acceptable system performance is indicated when the analysis is based on the small amplitude tests, chances are great that the stability of the system will be maintained at any amplitude. This is because the effects of dead band and saturation generally cause greater process loop stability. Oscillations below the dead band value are impossible because the loop gain is zero. The fact that the system is likely to become increasingly stable at higher amplitudes, is illustrated when it is considered that most process systems, when they are unstable, exhibit a limit cycle.

Standards have been published by various technical societies establishing recommended test amplitudes. One of the more popular standards recommends a 5% amplitude for pneumatic instruments unless this produces saturation, in which case the amplitude should be lowered to 1/2%. The standard also recommends that the test amplitude for positioner-actuators should be 5% unless saturation is encountered, in which case a 1/2% signal should be attempted and if this is impractical then any intermediate amplitude may be used.

Mean Level

The frequency response of most control devices is dependent upon the mean level about which the test is performed. However, the deviation in performance even between extreme operating points is normally not significant. It has, therefore, become standard practice to report only the data obtained at mid-range unless unusual behavior indicates special treatment is necessary.

Load

An entire group of problems arises when the load on a control device is considered. Some devices are highly sensitive to loading conditions so that test arrangements must closely duplicate actual service conditions. Other devices are relatively insensitive to loading which permits a certain amount of simplification in the test procedure and an increase in the generality of the data obtained. For this reason, and also because of the difference in the nature of the load on different devices, it is best to consider various classes of equipment separately.

Instruments with Electrical Outputs—Electronic instruments always have a loading specification which is as easy to duplicate in the laboratory as it is to provide in the field so that no problem ever arises.

Instruments with Pneumatic Outputs—The load imposed on a pneumatic instrument is a result of the receiver volume which the instrument must pressurize and the line which connects the instrument to this volume. The effect that the load produces on instrument performance must be explained by considering the means by which the instrument output is developed. Fig. 7 is a schematic illustration of a pneumatic instrument and its load. The instrument input may be any quantity "⊖" and the resulting output is a pressure, "P_1," that occurs at the instrument outlet or driving point.

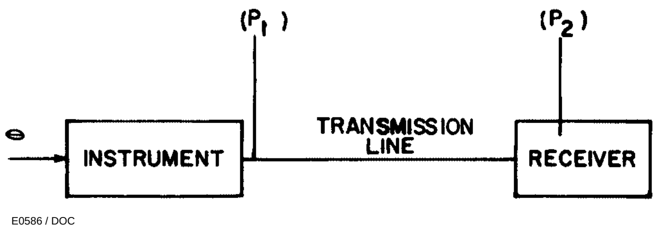


Figure 7. Schematic Representation of a Pneumatic Instrument and its Load

The output pressure is a result of the instrument creating a flow into or out of the load. The dynamic relationship between this flow and pressure is called the driving point impedance. A complete picture of the load as it appears to the instrument is given by the driving point impedance. For tubing lengths of not more than a few feet and which terminate in small volume receivers, the driving point impedance depends only on load volume. Thus the instrument response will not be influenced by load configuration. With increasing line length and receiver volume, the driving point impedance becomes a function of such dimensions as line diameter and length in addition to receiver volume. As line length is increased to beyond 150 ft., however, the driving point impedance approaches a constant value, completely independent of additional line length and of receiver volume. The instrument response, therefore, becomes identical for all long line loads.

Application of frequency response in the analysis of a process control system requires, in addition to the instrument response, the relation between the instrument output "P_1" and the receiver pressure "P_2". This function is usually called the output-input response of the load. For short lines terminating in small volume receivers, the output-input response is unity. Increasing line length and receiver volume causes the function to become increasingly significant. Unlike the driving point impedance, the output-input response of a long line remains a function of both configuration and volume. For most service conditions, the output-input response of the load can be calculated with acceptable accuracy, which is usually faster and more economical than obtaining the response experimentally.

Since the over-all response from instrument input to receiver pressure is the function required for system analysis, this relationship is sometimes measured directly in a single test. Although this is a perfectly valid approach, the relation between "⊖" and "P_2" is a unique function for each of an infinite number of possible loading conditions so that the limited generality of the driving point method is lost.

It should be pointed out that most of the standards published by the various technical societies as recommended testing practice require the instrument output to be measured at the driving point. When using data obtained in accordance with these standards, the output-input response of the load must not be neglected.

Pneumatic Actuators with Positioners—The basic problem in acquiring useful frequency response data on actuators arises from the widely varying and sometimes unpredictable loads to which they are subjected. There are three distinct types of loading to be considered, frictional loads from packing and seals, inertial loads from the mass of moving parts, and the forces resulting from fluid reactions on the valve plug as a result of fluid flow. The frictional load is quite variable and depends upon mechanical tolerances, surface condition of the parts, pressure differentials across the seal, lubricating properties of the fluid present, and length of service time. Actuators are often equipped with positioners to minimize the influence of packing friction in the control loop. Although the presence of friction does modify the dynamic performance of any positioner-actuator combination, a good positioner can do a fairly effective job. With a good positioner-actuator, the difference in response between conditions of no-load and normal packing friction load are of little significance. The variations in performance which result from normal changes in packing friction can almost always be neglected. On a positioner equipped piston actuator, internal friction occurring in the piston and stem seals is usually so large that any additional load from the bonnet packing will have an insignificant effect.

Inertial loads on actuators are variable because one size actuator may be used with a variety of valve plug styles and sizes. Fortunately, two factors help to eliminate any need for considering this problem. Because the valve plug is only a portion of the total inertial load, changes in plug size can have only a limited effect. Far more important, however, is the fact that the total inertial load on the actuator is always extremely small. The inertia in a typical actuator with a positioner can be increased from 2 to 3 times before any noticeable dynamic effects occur. For this reason, obtaining the response of a positioner-actuator without a valve plug attached is a justified test procedure.

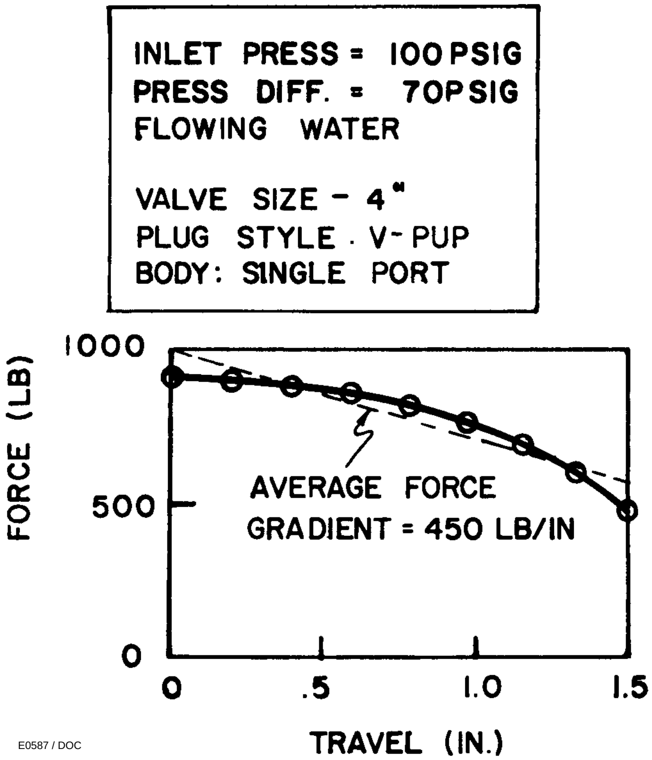


Figure 8. Typical Force Reaction on a Single Port Valve Due to Fluid Flow

Force reactions arising from fluid flow provide an additional actuator load to be considered. Fig. 8 shows this force as it occurs on a 4″ single port valve under conditions listed in the figure. Though actual values may differ considerably, the general shape of this curve is quite typical. The initial force present at zero valve lift is called static unbalance. This force has essentially no effect on dynamics, and is normally balanced out on diaphragm actuators by the initial compression set in the actuator spring. The slope of the curve in Fig. 8 shows that the force is a function of valve lift. This force gradient is very similar to the force gradient associated with mechanical springs which we define as the spring modulus, or rate, and it has the same influence on performance as would an additional actuator spring. For most double port valve applications and those single port valve applications where the pressure drop is very low, this flow force gradient is negligible compared to the modulus of the actuator spring. If, however, the service conditions for a particular application indicate that the flow force gradient will be significant, then the standard actuator spring is replaced with a special spring to compensate for the flow force gradient. Unless this gradient is highly non-linear, as might occur with an unusual valve design or extremely abnormal conditions, the response of an actuator with a standard spring under no-load conditions will not differ appreciably from a properly built actuator operating under actual flowing conditions.

It can be concluded that a positioner-actuator is relatively insensitive to loading conditions and that the no-load frequency response data is quite descriptive of the device during normal operation.

Actuators Without Positioners—For the usual actuator without a positioner, frequency response has absolutely no significance. In the special cases where this is not true, the response of the actuator alone is of minor importance compared to the loading effect it has on the instrument that drives it. Consequently, actuators should always be tested in combination with the driving instrument.

The loading effects on the actuator from valve inertia and fluid flow can be disregarded for the same reasons as were advanced in the previous section on positioner-actuators. The effect of packing friction, however, will almost always raise the dead band of the actuator above the saturation level of the driving instrument.

This overlap of dead band and saturation leaves no valid range of amplitude in which to perform a frequency response analysis. In the few cases where packing friction is very low and the saturation amplitude of the instrument is unusually high, some useful data may be obtained. In these cases it should be recognized that the pressure to motion response of the actuator is quite fast and will be of only minor importance. The dominant factor in the response of an instrument-actuator combination is the loading effect that the actuator imposes on its driving instrument. In the few cases where it is reasonable to consider the response of an actuator, it should be the instrument-actuator combination that is considered.

SUMMARY OF PART I

Experimental frequency response is an extremely valuable method of determining the dynamic behavior of individual control elements. With proper recognition of its limitations, the method can usually be used to evaluate individual control elements and to predict the behavior of complete control systems.

The frequency response of most control devices is affected by several details of the application. The Bode plot shown in Fig. 6 indicates all of the required information.

It is essential in acquiring meaningful data for a device that its input and output location are defined, that the input and output ranges and static gain are known, that complete load requirements are specified, especially for instrument devices, and that the energy level of the power supply is stated. The proper signal amplitude and mean level should be established by the experimenter, after the above information is determined.

PART II

LINEAR FUNCTIONS

The analysis of a control system requires knowledge of the dynamics of every element in the control loop. This information may be obtained by mathematical analysis but it is often expedient to acquire it experimentally. It is for this purpose that experimental frequency response finds its most valuable application. The ease with which an empirical transfer function can be derived from the Bode plot is a substantial factor in the choice of frequency response as a test method.

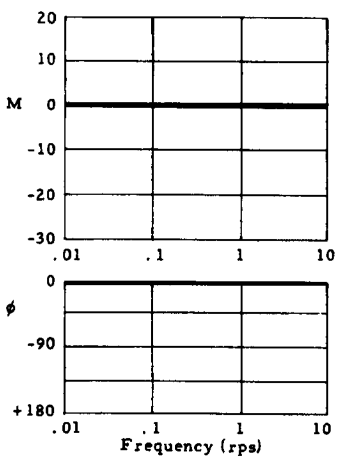
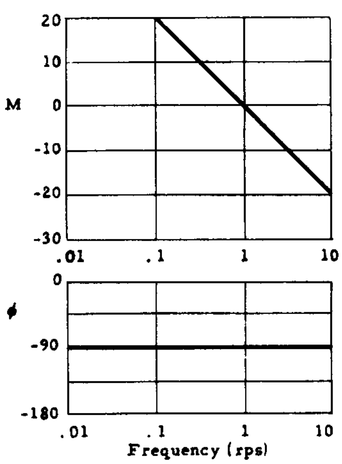
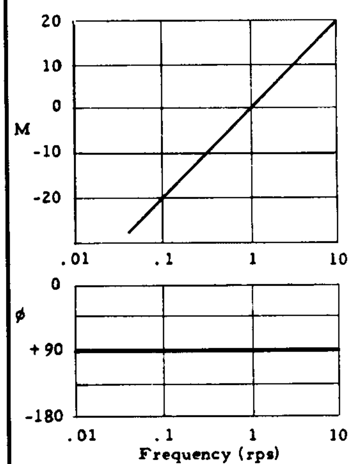
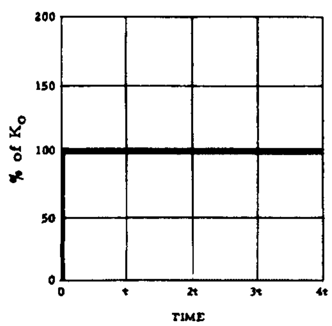
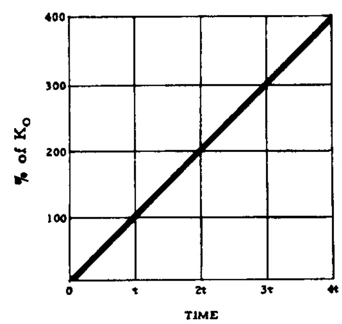
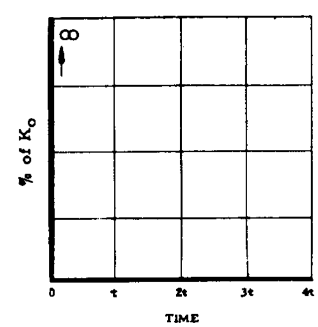
The frequency response of a device can be determined from its transfer function by substituting "jω" for the Laplace operator "s" and solving for the magnitude and phase angle of the resulting complex vector. If these vector equations are examined for various values of frequency, some interesting and useful characteristics may be observed. When plotted on logarithmic coordinates, the gain appears either exactly as a straight line or approaches straight line asymptotes. The intersections of these asymptotes are related in a convenient way to some of the coefficients in the transfer function. At high frequencies, the gain ultimately approaches an asymptote whose slope is directly proportional to the order of the transfer function of the device. The phase curve is not as easily approximated with straight line segments but it is found that the maximum phase shift that occurs is also related to the order of the transfer function. The relationships between the Bode charts and the transfer functions are illustrated in Table 1 for eight basic functions. Equations for the gain and phase shift are listed, the characteristic differential equation is shown and the response of the element to a unit step is illustrated.

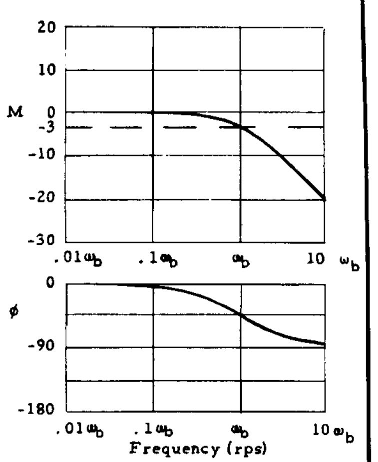
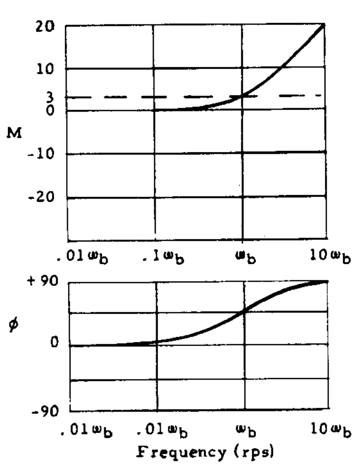
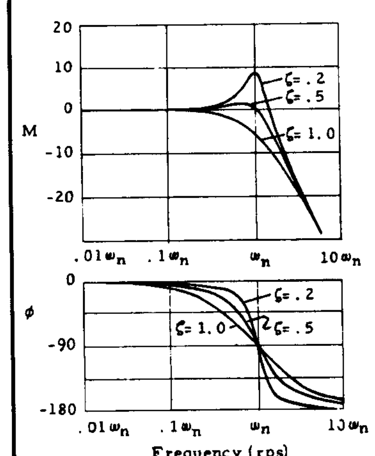
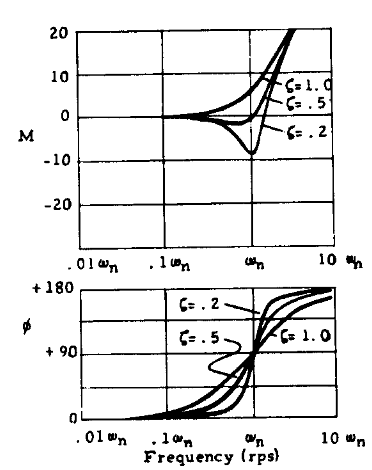
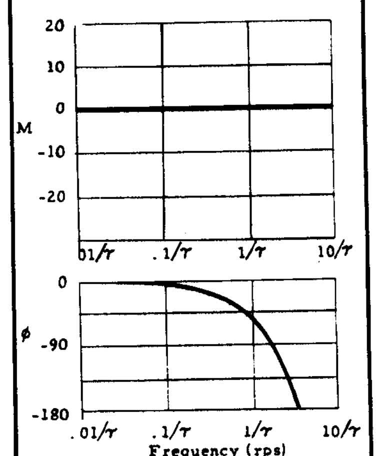
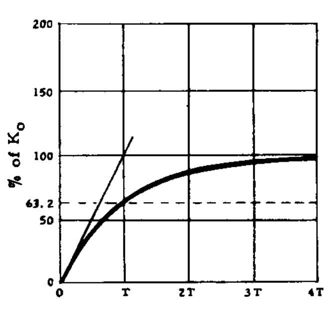
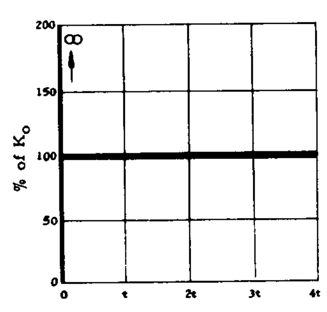
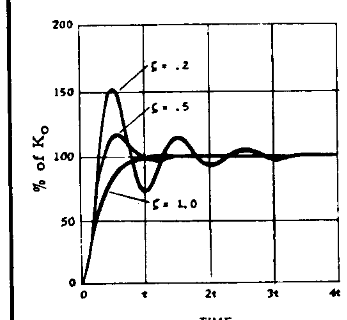
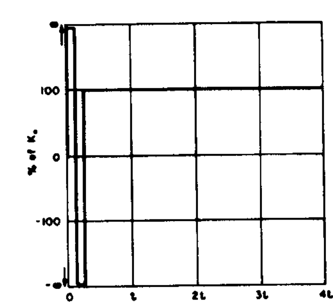
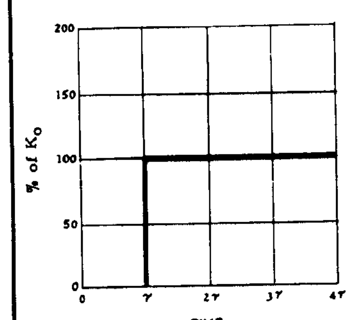
The relationships given in Table 1 can readily be applied to experimental data in order to arrive at an empirical transfer function. The eight basic elements, individually or in combination, can describe all of the usual control devices. The best procedure is to fit the gain curve first, using the asymptotes and their intersections to establish a trial function. If the phase shift of the trial function does not fit the test data, it will usually be due to the presence of a higher frequency element in the device whose only influence in the range of test frequencies is to add phase shift. Rather than complicate the transfer function with higher order terms, it is better to introduce the dead-time term even though it may be certain that no actual dead-time exists in the device. This technique will give the best low order empirical transfer function and will be completely valid throughout test frequency spectrum.

NON-LINEAR FUNCTIONS

No real elements can ever be completely described by linear equations, but most devices behave in a linear fashion within a certain limited operating range. Normally, these linear operating ranges permit very effective application of linear control theory but it is useful to be able to assess the effects of non-linearities as they creep in at the boundaries of the linear problem. Sinusoidal describing functions permit such an assessment and preserve the respectability of experimental frequency response under otherwise questionable conditions.

When a sinusoidal signal is applied to the input of a linear device, its output will be a sinusoid of the same frequency and may differ from the input only in amplitude and phase. The gain and phase curves used to describe this relationship are functions of the input frequency only. If a sinusoidal signal is applied to the input of a non-linear device, the output will be a nonsinusoidal periodic signal whose behavior is a function of both input frequency and amplitude. This output wave can be analyzed in terms of its Fourier components, however, and the fundamental harmonic will bear a relationship to the input sinusoid which can be described in gain and phase angle terms. The describing function is defined as the complex ratio of the fundamental component of the output signal to the input signal. All higher frequency components of the Fourier series are lumped together in an expression called the remnant. With this method the describing function and remnant form a quasi-linear system whose behavior is identical to the original non-linear system. The effect of the remnant is often negligible because it is composed of harmonics whose amplitudes are smaller than the fundamental and whose higher frequency signals usually encounter additional attenuation depending upon the filtering characteristics of succeeding elements. The describing function and remnant are valid only for the particular input signal for which they have been written, and both are amplitude dependent, therefore; they lack the generality of the transfer function used with linear elements. If, however, the remnant is clearly negligible and the fundamental harmonic can be shown to be relatively insensitive to amplitude, then the describing function approaches a completely linear transfer function. Dead band and saturation belong to a class of non-linearities whose sinusoidal describing functions are not frequency sensitive. They can be regarded merely as gain-changing elements. Figures 9 and 10 show the gain and phase angle of the sinusoidal describing functions for dead band and saturation as functions of input amplitude. Notice that when the amplitude of the non-linearity is small compared to the input amplitude, its presence is of little importance.

ELEMENT	GAIN ELEMENT	INTEGRATOR	DIFFERENTIATOR
TRANSFER FUNCTION	$F(s) = K_o$	$F(s) = \frac{K_o}{s}$	$F(s) = K_o s$
BODE PLOT NORMALIZED GAIN (db) PHASE SHIFT (DEGREES)			
RESPONSE TO A UNIT STEP INPUT AT TIME EQUAL ZERO.			
NORMALIZED GAIN EQUATION	$M = 1$	$M = \frac{1}{\omega}$	$M = \omega$
PHASE SHIFT EQUATION (RADIANS)	$\phi = 0$	$\phi = -\frac{\pi}{2}$	$\phi = +\frac{\pi}{2}$
CHARACTERISTIC DIFFERENTIAL EQUATION	$\theta_o = K_o\ \theta_i$	$\theta_o = K_o \int \theta_i dt$	$\theta_o = K_o \frac{d\theta_i}{dt}$
NOMENCLATURE E0651 / DOC	θ_i Input Variable θ_o Output Variable K_o Static Gain K Gain at any Frequency M Normalized Gain cps Cycles per Second rps Radians per Second		f Frequency (cps) ω Frequency (rps) f_b Break Frequency (cps) ω_b Break Frequency (rps) u Non-dimensionalized Frequency T Time Constant (sec) f_n Natural Frequency (cps) ω_n Natural Frequency (rps)

	FIRST ORDER LAG	FIRST ORDER LEAD	SECOND ORDER LAG	SECOND ORDER LEAD	DEAD-TIME
Transfer function	$F(s) = \frac{K_o}{Ts + 1}$	$F(s) = K_o (Ts + 1)$	$F(s) = \frac{K_o}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1}$	$F(s) = K_o \left[\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1\right]$	$F(s) = K_o e^{-\tau s}$
Frequency response					
Step response					
M	$M = \frac{1}{\sqrt{1 + (T\omega)^2}}$	$M = \sqrt{1 + (T\omega)^2}$	$M = \frac{1}{\sqrt{(1 - u^2)^2 + (2\zeta u)^2}}$	$M = \sqrt{(1 - u^2)^2 + (2\zeta u)^2}$	$M = 1$
φ	$\phi = \tan^{-1}(-\omega T)$	$\phi = \tan^{-1}(\omega T)$	$\phi = \tan^{-1} \frac{-2\zeta u}{1-u^2}$	$\phi = \tan^{-1} \frac{2\zeta u}{1-u^2}$	$\phi = -\omega\tau$
Differential equation	$T\frac{d\theta_o}{dt} + \theta_o = K_o \theta_i$	$\theta_o = TK_o \frac{d\theta_i}{dt} + K_o \theta_i$	$\left(\frac{1}{\omega_n}\right)^2 \frac{d^2\theta_o}{dt^2} + \left(\frac{2\zeta}{\omega_n}\right)\frac{d\theta_o}{dt} + \theta_o = K_o \theta_i$	$\theta_o = \left(\frac{K_o}{\omega_n^2}\right)\frac{d^2\theta_i}{dt^2} + \left(\frac{2\zeta K_o}{\omega_n}\right)\frac{d\theta_i}{dt} + K_o \theta_i$	$\theta_o = 0,\ t < \tau$ $\theta_o = K_o \theta_i,\ t \geq \tau$

ζ Damping Ratio
τ Dead-time (sec)
t Unit Time (sec)
ϕ Phase Shift in Degrees or Radians as Specified
F(s) Transfer Function Defined as, $F(s) = \frac{\mathcal{L}(\theta_o)}{\mathcal{L}(\theta_i)}$

Conversion Equations

$u = \frac{f}{f_n} = \frac{\omega}{\omega_n}$

$\omega = 2\pi f$

$T = \frac{1}{\omega_b} = \frac{1}{2\pi f_b}$

$M = \frac{K}{K_o}$

1 radian = 57.3 degrees

DECIBEL CONVERSION TABLE

$x = 20 \log_{10} N$

N	0	1	2	3	4	5	6	7	8	9
0.0	x -	-40.00	-33.98	-30.46	-27.96	-26.02	-24.44	-23.10	-21.94	-20.92
0.1	-20.00	-19.17	-18.42	-17.72	-17.08	-16.48	-15.92	-15.39	-14.89	-14.42
0.2	-13.98	-13.56	-13.15	-12.77	-12.40	-12.04	-11.70	-11.37	-11.06	-10.76
0.3	-10.46	-10.17	-9.90	-9.63	-9.37	-9.12	-8.87	-8.64	-8.40	-8.18
0.4	-7.96	-7.74	-7.54	-7.33	-7.13	-6.94	-6.74	-6.56	-6.38	-6.20
0.5	-6.02	-5.85	-5.68	-5.51	-5.35	-5.19	-5.04	-4.88	-4.73	-4.58
0.6	-4.44	-4.29	-4.15	-4.01	-3.88	-3.74	-3.61	-3.48	-3.35	-3.22
0.7	-3.10	-2.97	-2.85	-2.73	-2.62	-2.50	-2.38	-2.27	-2.16	-2.05
0.8	-1.94	-1.83	-1.72	-1.62	-1.51	-1.41	-1.31	-1.21	-1.11	-1.01
0.9	-0.92	-0.82	-0.72	-0.63	-0.54	-0.45	-0.35	-0.26	-0.18	-0.09
1.0	0.00	0.09	0.17	0.26	0.34	0.42	0.51	0.59	0.67	0.75
1.1	0.83	0.91	0.98	1.06	1.14	1.21	1.29	1.36	1.44	1.51
1.2	1.58	1.66	1.73	1.80	1.87	1.94	2.01	2.08	2.14	2.21
1.3	2.28	2.35	2.41	2.48	2.54	2.61	2.67	2.73	2.80	2.86
1.4	2.92	2.98	3.05	3.11	3.17	3.23	3.29	3.35	3.41	3.46
1.5	3.52	3.58	3.64	3.69	3.75	3.81	3.86	3.92	3.97	4.03
1.6	4.08	4.14	4.19	4.24	4.30	4.35	4.40	4.45	4.51	4.56
1.7	4.61	4.66	4.71	4.76	4.81	4.86	4.91	4.96	5.01	5.06
1.8	5.11	5.15	5.20	5.25	5.30	5.34	5.39	5.44	5.48	5.53
1.9	5.58	5.62	5.67	5.71	5.76	5.80	5.85	5.89	5.93	5.98
2.	6.02	6.44	6.85	7.23	7.60	7.96	8.30	8.63	8.94	9.25
3.	9.54	9.83	10.10	10.37	10.63	10.88	11.13	11.36	11.60	11.82
4.	12.04	12.26	12.46	12.67	12.87	13.06	13.26	13.44	13.62	13.80
5.	13.98	14.15	14.32	14.49	14.65	14.81	14.96	15.12	15.27	15.42
6.	15.56	15.71	15.85	15.99	16.12	16.26	16.39	16.52	16.65	16.78
7.	16.90	17.03	17.15	17.27	17.38	17.50	17.62	17.73	17.84	17.95
8.	18.06	18.17	18.28	18.38	18.49	18.59	18.69	18.79	18.89	18.99
9.	19.08	19.18	19.28	19.37	19.46	19.55	19.65	19.74	19.82	19.91
	0	1	2	3	4	5	6	7	8	9

DECIBEL CONVERSION TABLE

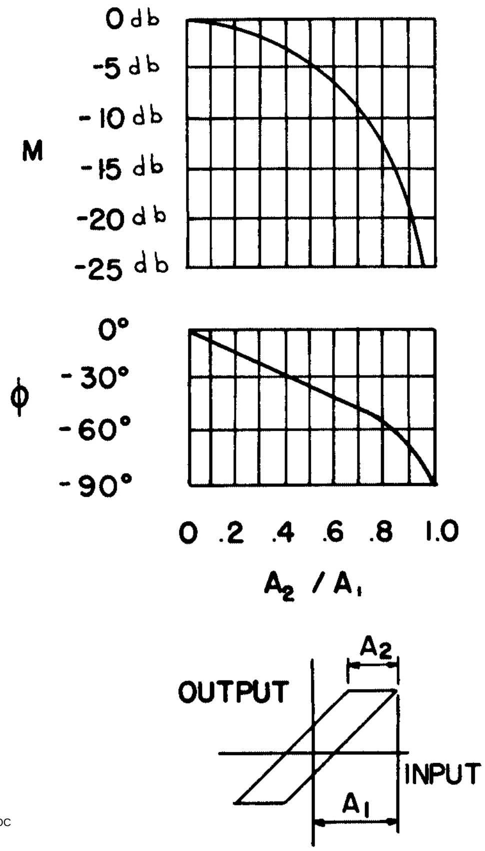


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Figure 9. Describing Function for Dead Band

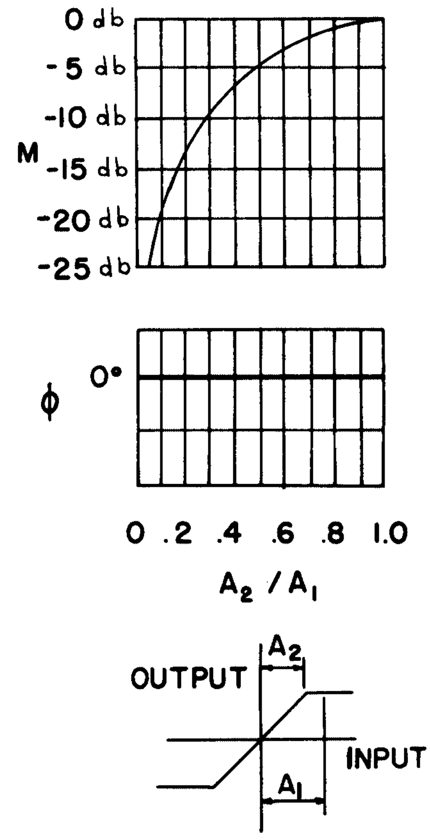

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Figure 10. Describing Function for Saturation or Limiting

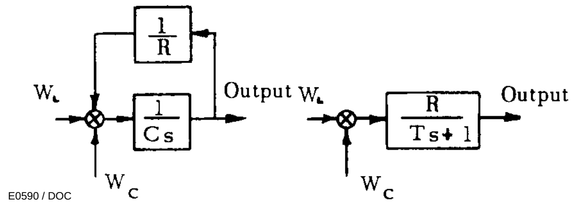
Modern sinusoidal frequency analyzers are capable of performing a true Fourier analysis on an output signal and automatically computing the gain and phase relationships based on the fundamental harmonic. This technique, coupled with the proper choice of test signal amplitude, strongly supports the usefulness and propriety of experimental frequency response as a means of determining the dynamics of process control equipment.

RESPONSE OF SOME TYPICAL CONTROL ELEMENTS

Some of the elements which appear frequently in control loops can be conveniently handled by theoretical equations or by empirically modified theoretical equations. In this concluding section of the manual, equations are provided for several of these elements along with a few application guides.

Pressure and Liquid Level Processes

The processes discussed here are energy storage systems. The output in each case is defined as the variable which is to be controlled, the input is the variable on which the output depends. In these processes the controlled variable is the change in energy level manifested by changes in pressure or head, which is a function of the net accumulation of fluid within the process chamber or net inflow. If the energy level varies from one location to another within the system, the process dynamics become a fairly complicated function of the distributed system parameters; specifically, this situation arises in the case of long transmission lines. This problem has been studied thoroughly by Schuder and Blunck[1] and others[2] but will not be discussed here. A great many energy storage processes, however, occur within relatively short pipe sections or tanks where the parameters can be lumped. For these simpler cases, the energy level is proportional to the time integral of the net flow into the system. Since variations in the energy level may influence the net flow, some negative feedback or self-regulation can occur around this integrator. The process transfer function becomes a first order lag when this feedback is significant, otherwise it remains an integrator.[3]



C	= Process capacitance
R	= Process resistance
W_L	= Load flow
W_C	= Control valve flow
W_N	= Net flow, ($W_N = W_L - W_C$)
V	= Volume of process vessel
P_C	= Process output (pressure)
ΔP_1	= Pressure loss at inlet
ΔH_1	= Head loss at inlet
ΔP_0	= Pressure loss at outlet
ΔH_0	= Head loss at outlet
A	= Surface area of liquid level
w	= Specific weight
c	= Sonic velocity in process fluid
g	= Gravitational constant
β	= Bulk modulus of process liquid—should be modified to account for vessel elasticity and gas entrainment

A bar over any term indicates the average steady-state value of that variable.

Liquid Level—The majority of liquid level processes have negligible self-regulation. A useful guide is: if the process time constant is the largest lag in the loop and if it is larger than the next smaller lag by a factor of ten or more, the process may be treated as an integrator. Convenient units are: head in Ft., flow in Ft.^{3}/sec, and area in Ft.^{2}.

$$C = A, 1/R = \frac{\overline{W}_L}{2}\left[\frac{1}{\Delta\overline{H}_1} + \frac{1}{\Delta\overline{H}_2}\right], T = RC$$

Liquid Pressure—The time constant of a liquid pressure process is usually so small that the process dynamics can be ignored and the transfer function reduces to that of a simple gain element. Convenient units are: volume in in.^{3}, specific weight in lb./in.^{3}, bulk modulus in psi, flow in lb./sec. and pressure in psi.

$$C = \frac{Vw}{\beta}, 1/R = \frac{\overline{W}_L}{2}\left[\frac{1}{\Delta P_1} + \frac{1}{\Delta P_0}\right], T = RC$$

Gas Pressure—A very wide range of possible time constants can occur in the various commonly found gas pressure process conditions. The process dynamics are nearly always significant, however, and usually require a first order lag transfer function. Convenient units are: volume in in.^{3}, velocity in in./sec,

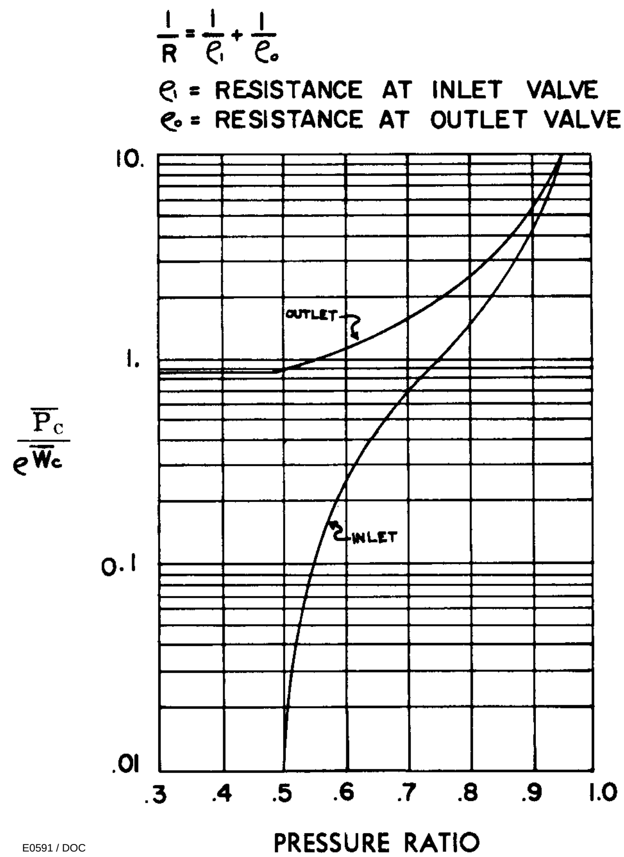


Figure 11. Resistances for Pressure Processes

flow in lb./sec, pressure in psi and the gravitational constant in in./sec^2.

$$C = \frac{V_g}{c^2} \qquad \text{for } 1/R \textit{ see Fig. } 12$$

Process Controllers

Process controllers may provide proportional, integral and derivative response actions in various combinations. The controller frequency response is normally measured for proportional action only. The effect of the integral and derivative modes on response is illustrated in Fig. 12. Mode settings at frequencies in excess of the controller bandwidth do not produce the intended effect and are generally not useful.

Integral or reset action provides very high static gain to reduce the steady-state process offset. It is desirable to use the fastest reset setting possible without introducing amplification or excessive phase shift at the system operating frequency. Derivative or rate action increases the operating frequency of the process loop, permitting faster recovery and also allowing stable behavior at higher gain and faster reset settings.

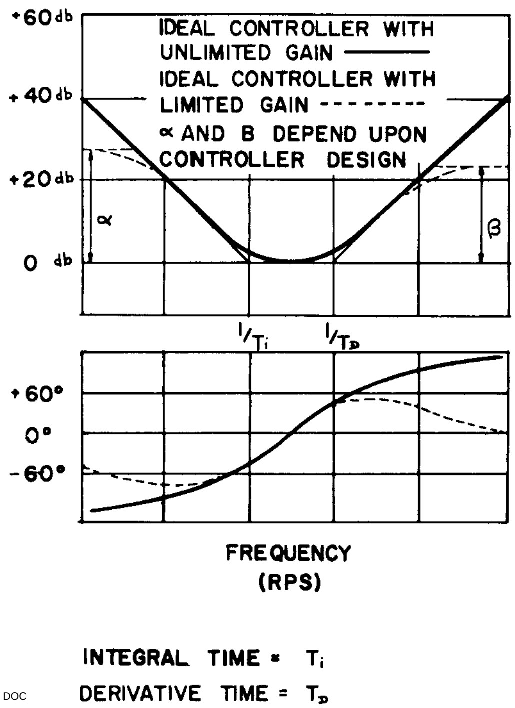


Figure 12. Effect of Controller Modes on Frequency Response

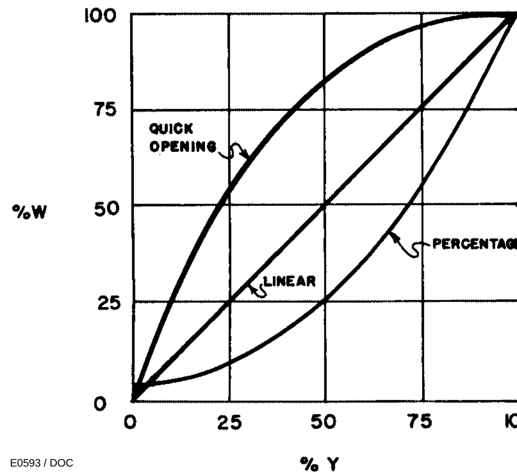


Figure 13. Typical Valve Characteristics

Several methods are used as guides toward finding optimum controller settings but all are approximate because certain limiting assumptions must be made about the process and other system elements. One reasonable approach is called the ultimate gain method. If %P B_m is the narrowest proportional band setting which just provides incipient instability and if N is the period of oscillation, then good controller settings are approximately:

%PB = 2(%P B_m)
Reset time = 1/2 (N)
Rate time = 1/10 (N)

Valve Response

The reaction between valve motion and flow at the valve is fast enough that the valve dynamics are never of interest in process control. The transfer function, obviously, is just a simple gain element. The valve gain can be computed from the following relationship where "m" is the slope of the characteristic curve in Fig. 13, "W" is flow and "y" is valve lift.

$$G_v = m\left(\frac{W_{max}}{y_{max}}\right)$$

If the pressure differential across the control valve remains relatively constant, then the ordinate in Fig. 13 can be based on percent of C_V and equations can be written to replace "m" for at least two standard valve characteristics.

$$\text{for } \textit{linear valves},\ m = 1$$

$$\text{for } \textit{percentage valves},\ m = \left(\frac{W}{W_{max}}\right) 1n\,R$$

where "1n R" is the natural logarithm of the valve rangeability.

Selection of the proper control valve characteristic will provide uniform system stability regardless of the operating point. In general, very slow processes such as liquid level will require a linear valve characteristic and very fast processes such as liquid pressure require a percentage valve characteristic. Intermediate cases and those having varying control valve differentials should be evaluated according to their individual requirements.

Response of Instrument Transmission Lines

The analysis of a pneumatic instrument transmission line is complicated by the distribution of parameters within the system. For a complete study of this problem, see references 1, 2 and 4. Because these elements often contribute significantly to the dynamics of a process control loop, several simplified approximations to the problem have been proposed. The equation given here has proven satisfactory in applications where the phase shift from the transmission line does not exceed 150° at the operating frequency of the loop.[4] It also is restricted to 3/16″ I.D. copper tubing terminating in a 1.2 in.^{3} receiver volume with an instrument signal range of 3-15 psig.

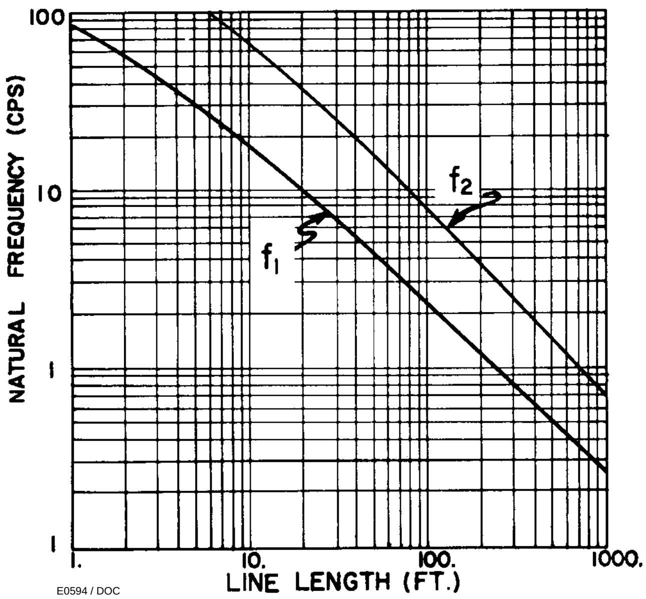


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Figure 14. Coefficients for Transfer Function of 1/4” Tubing Transmitting Air at a Mean Pressure of 9 psig, Mean Temperature of 70 °F into a Terminal Volume 1.2 Cubic Inches

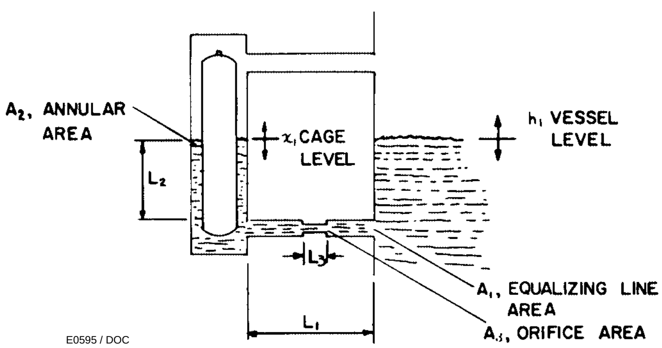

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Figure 15. Caged Level Sensing Element

$$\frac{(P_2)}{\mathcal{L}(P_1)} = \frac{1}{\left[\left(\frac{s}{\omega_1}\right)^2 + \frac{2\zeta_1 s}{\omega_1} + 1\right]\left[\left(\frac{s}{\omega_2}\right)^2 + \frac{2\zeta_2 s}{\omega_2} + 1\right]}$$

$$\zeta_1 = \frac{10.6}{\omega_1} \qquad \zeta_2 = \frac{10.6}{\omega_2}$$

The coefficients “ω_1” and “ω_2” can be evaluated from the chart in Fig. 14.

Response of Caged Level Sensing Elements

A cage mounted float or displacer is a very common sensing device for liquid level control applications. Although its relative low cost and good reliability make it a popular control element, thoughtless installation may result in rather undesirable dynamic behavior. Referring to Fig. 15, the relationship between level “x” in the float cage and the process level “h” is given by the underdamped second order transfer function:

$$\frac{\mathcal{L}(x)}{\mathcal{L}(h)} = \frac{1}{\left(\frac{s}{\omega_n}\right)^2 + \frac{2\zeta s}{\omega_n} + 1}$$

If smooth, unrestricted equalizing lines are used between the process vessel and float cage, damping ratios of nearly .05 are obtained with most fluids. Because this element is usually quite slow and because the process is usually an integrator, the operating frequency of the process loop is very likely to occur right at the resonant peak of this liquid column. The overall process control will be substantially improved by any modification which will increase “ω_n” (thus permitting greater attenuation by the integrator) or reduce the resonant peak by increasing “ζ”.

The introduction of a thin flat plate orifice into the lower equalizing line is a very effective damping device. Damping ratios have been increased from .05 to over .3 by the use of a 1/2″ orifice in a 2″ diameter equalizing line without significantly reducing natural frequency. Since maintaining a high natural frequency is also an important factor in the problem, it is useful to examine the equation that approximately defines this parameter:

$$\omega_n = \sqrt{\frac{g}{L_2 + L_1\left(\frac{A_2}{A_1}\right) + L_3\left(\frac{A_2}{A_3}\right)}}$$

where “g” is the gravitational constant and the other terms are defined in Fig. 15. Inspection of this equation reveals the importance of keeping the equalizing lines short with relatively large diameters and using a thin damping orifice.

The displacer itself is a highly underdamped second order element but its natural frequency is sufficiently higher than that of the liquid column that its dynamics can always be neglected.

Response of Valve Actuators

The dynamics of a valve actuator usually have a significant influence on process loop performance. Because a certain amount of choice is available in the selection of an actuator, some attempt should be made to determine what constitutes desirable actuator performance. The relative merits of slow versus fast response and the effect of damping are the factors to be considered.

Fig. 16 is a block diagram of a typical process loop. Notice that all of the control elements appear in the feedback path of the system. To provide good process regulation these elements should exhibit fast response

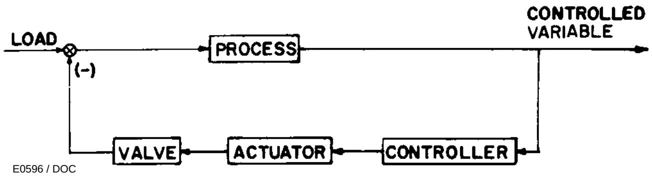

Figure 16. Block Diagram for a General Process Control

and high gain but this is often an incompatible combination. Most high gain control loops are stabilized by a single dominant time lag. If the process provides this lag, then the desirability of a fast actuator is clearly established. Increasing the response of the feedback elements will improve the overall process control in every respect. However, when the process is relatively fast, high gain can be achieved only when a dominant lag appears in the feedback path. The location of the dominant lag does not affect the steady-state process error but it greatly influences transient response. Lags in the feedback path reduce the ability of the system to cope with rapid load changes by producing abnormally large initial overshoots and by extending the recovery time. This condition becomes most acute with very fast processes such as liquid pressure control where all the significant dynamics of the loop appear in the feedback path. In this case, use of a low response actuator will result in good steady-state control with very poor transient response. Substitution of a high response actuator will improve the transient behavior but will also increase the steady-state error. The best control is achieved by using the fastest possible feedback elements and by providing integral or reset action in the controller. This combination requires low gain or broad proportional band settings but good steady-state accuracy is obtained from the reset mode. The dominant lag of the system may be considered to be the reset mode and, being adjustable, permits the loop to be "tuned" for the best possible transient response.

In some cases a broad proportional band reset controller may not be available. In these cases, if the transient requirements can be relaxed because of very slow and gradual load changes, then a low response actuator may be the most favorable choice. It should be emphasized, however, that a slow actuator may be preferred only when a broad proportional band reset controller is not available and when the process load changes are extremely mild.

The response curve for an actuator should indicate a damping ratio in the range of 0.5 to 1.0 although somewhat greater damping does not seriously impair the process loop performance. Damping ratios of less than 0.5 would not be objectionable if the actuator were among the fastest elements in the loop. Since this is seldom the case, such underdamping should be avoided.

CONCLUSION

The analysis of a control system requires knowledge of the dynamics of every element in the control loop. For many control devices, experimental frequency response provides the most convenient means of obtaining the required dynamic expression. Application of this data must always be made with an awareness of the associated limitations and restrictions. Many of these qualifying limitations are well known, some are not. It has been the intention in this manual to clarify the significant points of the problem with the hope of increasing the utility of experimental frequency response data in the industrial process control field.

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D350401X012 / Printed in U.S.A. / 1969

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