ABSTRACT

Over the years the functionality and performance of instrumentation used with head type flowmeters has steadily improved. The technology employed in pressure instrumentation has resulted in a migration from mechanical/pneumatic transmitter to analog electronic transmitters to microprocessor based smart electronic transmitters. Similar improvements have occurred in temperature instrumentation. The logical next step in this progression is the consolidation of multiple process measurements in a single transmitter package. With all of the process measurements available, the benefits to the user can be further enhanced by performing flow calculations within the transmitter.

The benefits of using the process variables to continuously calculate the terms in the equation for flow through a head type meter are presented. A comparison is made between this method and the commonly used method in which many terms are considered to be constant. Significant improvements can be realized by the real-time calculation of these terms, particularly the discharge coefficient and fluid density.

INTRODUCTION

The venerable orifice meter has enjoyed a long and versatile life dating back to Roman times. During its life, the orifice meter has seen the advent of many technological advances which have enhanced its usefulness. The consistent goal of these advances was to improve the overall accuracy and usable range of the orifice meter. The current state of the art in pressure instrumentation affords us options that would have been inconceivable to our imperial predecessors and only dreamed of a matter of years ago. The objective of this paper is to show how the current state of the art in transmitters can improve the accuracy and rangeability of orifice flow meters.
FLOW EQUATION FOR A DIFFERENTIAL PRODUCER

The equation describing the flow of liquids and gases through a differential producer such as an orifice meter can be written as:

\[ Q_m = NC_dEY_1d^2 \sqrt{\rho h} \]

where:
- \( Q_m \) = Mass flow rate (dimensions of mass per unit time)
- \( N \) = Units conversion factor (dimensionless)
- \( C_d \) = Discharge coefficient (dimensionless)
- \( E \) = Velocity of approach coefficient (dimensionless)
- \( Y_1 \) = Gas expansion factor (dimensionless)
- \( d \) = Orifice bore (dimensions of length)
- \( \rho \) = Fluid density (dimensions of mass per unit volume)
- \( h \) = Differential pressure (dimensions of force per unit area - typically given in inches of water at a specified temperature)

The velocity of approach factor term, \( E \), accounts for changes in fluid velocity as it passes through the orifice. It is expressed as

\[ E = \frac{1}{\sqrt{1 - \beta^4}}, \]

where the beta ratio, \( \beta = \frac{d}{D} \), is the ratio of the orifice bore to the meter tube bore. The gas expansion factor term, \( Y_1 \), accounts for changes in fluid density as it passes through the orifice. For incompressible fluids in which there is no density change, the gas expansion factor term has a value of 1.0. The gas expansion factor term will be discussed in more detail below.

A common practice is to assume that many of these terms are constant and approximate the flow equation as:

\[ Q_m = K_1 \sqrt{h} \]

For gas applications, a slightly more sophisticated approximation can be written as:

\[ Q_m = K_2 \frac{hP}{T} \]

These simplifications to the flow equation are often done out of convenience. In typical applications, the process variables measured using individual transmitters are sent to a control room where they may be used to calculate the flow rate in a distributed control system. In cases where the only process variable measured is the differential pressure, there is no recourse but to use the simplest approximation of the flow equation. In cases where the pressure and temperature are available, the flow equation may still be approximated because of its attractive simplicity and the desire to minimize the use of expensive computer resources. Furthermore, in this era of lean organization structure, the expertise required to implement the full flow equation may no longer be present in-house. This can necessitate the hiring of expensive consultants when the need to add new flow measurement points or to upgrade existing points arises.

Approximating the flow equation as described above can result in a significant flow measurement error because of variations in the terms that are assumed to be constant. Variations in the discharge coefficient and fluid density have the most significant effect. In addition, for gas flow applications, the variations in the gas expansion factor term can also result in significant errors in flow rate when it is assumed to be constant.

To evaluate the effects of simplifying the flow equation, it is instructive to look at variations of the parameters in the flow equation for a gas and a liquid application. The flow equation will be evaluated for air flow with pressure and temperature variations of 100 - 200 psia and 80 - 120 degrees F and for water flow with temperature variations of 80 - 120 degrees F. Both cases will be evaluated for a 2-inch orifice meter run with a beta ratio of 0.6. Throughout this analysis it will be assumed that the orifice meter is in full compliance with the requirements of existing domestic and/or international standards pertaining to orifice meters.
Discharge Coefficient

The discharge coefficient is an empirical term describing the amount of flow that actually passes through the orifice meter. It corrects the theoretical flow equation for the effects of friction, velocity profile of the fluid in the pipe, and the location of the pressure taps. Through the years, much effort has gone into quantifying the discharge coefficient of the square edged orifice plate, resulting in complicated equations that are functions of Reynolds number and beta ratio. The ISO/ASME discharge coefficient equation[1] for a 2-inch orifice meter with flange taps is given by the expression at the bottom of the page(1):

\[ C_d = 0.5959 + 0.0315\beta^{2.1} - 0.184\beta^8 + 0.039\beta^4 \left(1 - \beta^4\right)^{-1} - 0.0337D^{-1}\beta^3 + 91.71\beta^{2.5}R_D^{-0.75} \]

where:
- \( C_d \) = Discharge coefficient (dimensionless)
- \( R_D \) = Reynolds number based on meter tube diameter (dimensionless)
- \( \beta \) = Beta ratio (orifice bore/meter tube bore, dimensionless)
- \( D \) = Meter tube diameter (inches)

The Reynolds number is defined as:
\[ R_D = \frac{22738Q_m}{\mu D} \]

where:
- \( Q_m \) = Mass flow rate (units of pounds per second)
- \( D \) = Meter tube diameter (units of inches)
- \( \mu \) = Fluid viscosity (units of centipoise)

For other tap locations and meter tube diameters greater than 2.3 inches, other discharge coefficient equations have been developed. In addition, other expressions for the discharge coefficient of square edged orifice meters are given by the Reader-Harris/Gallagher equation in A.G.A Report No. 3[2].

Because the discharge coefficient is a function of flow rate, it must be calculated iteratively. This can consume valuable control room computer resources and complicates the determination of the “constant” values to be used in the simplified version of the flow equation.

The assumption that the discharge coefficient had a constant value of 0.6 would result in bias errors on the order of 1.5-3.5% for the water flow example and 1% for the gas flow example. These errors can be reduced by calculating a more representative constant discharge coefficient value. In these examples, there are still discharge coefficient variations of up to 2% for the water flow and 0.5% for the air flow which will show up as errors in flow rate.

FIGURE 1. Discharge Coefficient

The discharge coefficient for an orifice meter in a 2-inch pipe is plotted as a function of Reynolds number for several beta ratios in Figure 1. It can be seen that the discharge coefficient asymptotically approaches a value of approximately 0.6 for very high Reynolds numbers, but that it deviates from this value as the Reynolds number is decreased. In most industrial flow applications the Reynolds numbers are much smaller than those for which the discharge coefficient can reasonably be assumed to have a value of 0.6. For a flow range of 8:1 (i.e., 64:1 range in differential pressure) the Reynolds numbers for the air flow example ranged from 76,230 - 924,000 and for the water flow example from 20,400 - 245,340.
Fluid Density

For the pressures typically encountered in the process industry the density of liquids is a function of temperature. The density of gases is a function of both pressure and temperature. For these example cases the density of air was calculated using the AIChE equation of state given by the equations at the bottom of the page:

\[
\rho = \frac{P}{\rho_{\text{mol}}^2 B^2} \quad \rho_{\text{mol}} = \frac{1}{\rho_{\text{mol}}} \quad B = a + \frac{b}{T_k} + \frac{c}{T_k^2} + \frac{d}{T_k^3} + \frac{e}{T_k^4}
\]

where \( \rho \) is the molar density in units of kg/mol, the pressure and temperature are in Pascals and degrees K, respectively, \( M_W \) is the molecular weight, \( B \) is the second virial coefficient and the density is in units of kg/m\(^3\). The values of the constants \( a \) - \( e \) are:

\[
\begin{align*}
a &= 4.3045 \times 10^{-2} \\
b &= -1.7121 \times 10^1 \\
c &= 1.7131 \times 10^5 \\
d &= -3.4138 \times 10^{15} \\
e &= 3.0380 \times 10^{17}
\end{align*}
\]

The water density was calculated using the PTB equation given as:

\[
\rho_i = \frac{5}{999.8395639 \sum_{i=0}^{9} a_i \theta_i}
\]

where the density is in units of kg/m\(^3\) and the temperature is in degrees C.

The constants are:

\[
\begin{align*}
a_0 &= 999.8395639 \\
a_1 &= 6.798299989 \times 10^{-2} \\
a_2 &= -9.106025564 \times 10^{-3} \\
a_3 &= 1.0052729998 \times 10^{-4} \\
a_4 &= 1.126713526 \times 10^{-6} \\
a_5 &= 6.5917956066 \times 10^{-9}
\end{align*}
\]

Using these equations, the density variations that result over the operating range of pressure and temperature for the air flow example are as high as 40%. For the water flow example, the density variations over the temperature range is 0.8%.

Gas Expansion Factor

The density of a gas changes as it flows through an orifice. This is accounted for by the gas expansion factor term in the flow equation. For smoothly contoured devices such as nozzles or venturi meters the adiabatic gas expansion factor is used. For square edged orifices the gas expansion factor is an empirical term accounting for the changes in gas density as the flow goes through the orifice meter. The expression for the gas expansion factor is given by the Buckingham equation:

\[
Y_1 = 1 - (0.41 + 0.35 \beta^4) \left( \frac{h}{27.73 P} \right)
\]

where \( \kappa \) is the isentropic exponent of the gas. Recent work by Kinghorn and Seidl may lead to changes in the expansion factor expression. They Buckingham equation was used in this analysis. In the air flow example and an 8:1 range in flow rate results in a change in the gas expansion factor changes of approximately 2.5%.

Temperature Effects

Thermal expansion affects the orifice bore. For dissimilar orifice plate and meter tube materials, the velocity of approach factor is also affected by thermal expansion. For a carbon steel meter tube and 316 SST orifice plate and for the temperature variations assumed in these examples, the variations in the \( d^2 \) and \( E \) terms are 0.07% and 0.02%, respectively. There is also a negligible effect of temperature on the gas expansion factor term.

Temperature changes affect the discharge coefficient by virtue of changes in the fluid viscosity. This effect is larger for liquid flow cases where the Reynolds numbers are smaller. For the water flow example, the change in viscosity over the temperature range is approximately 54% resulting in a change in discharge coefficient on the order of 0.5%. For the air flow example, the change in viscosity over the temperature range is approximately 5.6% resulting in a change in discharge coefficient of less than 0.1%. The viscosity of water was calculated using.

\[
\mu = \frac{a_0 + a_1 \theta + a_2 \theta^2 + a_3 \theta^3}{1 \times 10^3}
\]

where the constants are:

\[
\begin{align*}
a_0 &= 7.25 \times 10^{-4} \\
a_1 &= 3.0 \times 10^{-5} \\
a_2 &= -8.0 \times 10^{-8} \\
a_3 &= 0.0123
\end{align*}
\]


\[ \mu = 1.0019 \times 10^{-6} \] where

\[ R = \frac{-1.3272(T - 20) - 0.001053(T - 20)^2}{T + 105}; \mu \text{ in cP and } T \text{ in degrees C.} \]

The viscosity of air was calculated using the AIChE vapor viscosity equation \[11\]:

\[ \mu = \frac{aT^b}{1 + \frac{c}{T_k} + \frac{d}{T_k^2}}; \mu \text{ in Pa-sec and } T_k \text{ is degrees K.} \]

The values of the constants a-d are:

a=1.4373e-6; b=5.023e-1; c=1.08e2; d=0.0.

**Advantages of multiple process variable transmitters**

As the discussion above has shown, simplifying the flow equation by assuming that the constituent terms are constant can result in large errors in flow rate. The only way to eliminate these errors is by the continuous calculation of such terms as the discharge coefficient, fluid density, gas expansion factor, and fluid viscosity. A multiple process variable transmitter with continuous on-board calculation of the mass flow rate, including all of the terms in the flow equation, offers the significant advantage of enhancing the information available from a given measurement point. Whereas traditionally such measurement points provided only process variable information, the same measurement point can now provide the actual parameter of interest, the mass flow rate, in addition to the process variables. This has the added benefit of reducing the calculation load on the distributed control system. Transmitters containing differential pressure, pressure, and temperature sensors in a single housing offer a number of other obvious advantages. These include lower installation costs, a reduced number of process penetrations, and fewer potential leak paths that require monitoring for the fugitive emissions.

The following section will discuss the uncertainties associated with measuring flow with an orifice meter. Sample uncertainty calculations will be given using actual transmitter performance data which will demonstrate that the usable range for orifice meters can be extended far beyond the 3:1 range that has been traditionally accepted.

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**RANGEABILITY OF ORIFICE METERS**

Conventional wisdom has held that, due to the range down performance of the differential pressure transmitters, the effective range of an orifice meter is limited to 3:1 (9:1 in differential pressure). To evaluate increasing the rangeability of the orifice meter, test data from four Multivariable™ transmitters was used in flow uncertainty calculations. By measuring the performance of typical transmitters, it can be shown that over a wide range of flow rates, the differential pressure is not a large contributor to the overall flow uncertainty.

**Flow Uncertainty**

ASME MFC-3M-1989\[12\] defines uncertainty as "a range of values within which the true value of the measurement is estimated to lie at the 95% probability." The equation for calculating flow uncertainty is stated below\(1\):

\[ \frac{\delta Q_m}{Q_m} = \left[ \frac{\delta C_d}{C_d} \right]^2 + \left[ \frac{\delta d}{d} \right]^2 + \left[ \frac{2\beta^4}{1-\beta^2} \right] \left[ \frac{\delta D}{D} \right]^2 + \left[ \frac{2}{1-\beta^2} \right] \left[ 2 \gamma \right] \left[ \frac{\delta h}{h} \right]^2 + \left[ \frac{2\delta h}{h} \right]^2 + \left[ \frac{2\delta P}{P} \right]^2 \]  

\(\delta Y_1 = 0.144P\) where \(h\) is in units of inches of water and \(P\) is in units of psia.

As the ratio of differential pressure to pressure decreases the uncertainty of the gas expansion factor decreases. The expansion factor for incompressible fluids (liquids) is a constant of 1, and has no uncertainty.
The uncertainty in the meter tube and orifice bores are caused by tolerances in the physical dimensions. ASME MFC-3M-1989 maximum values of 0.4% for meter tube bore uncertainty and 0.07% for orifice bore uncertainty and were used in the calculations.

Density Uncertainty

Gas Density can be rewritten as:

\[ \rho = \frac{144PM_w}{ZRT} \]

where \( P \) is in psia, \( M_w \) (molecular weight) is in lbm/mol, \( T \) is in degrees, \( Z \) (compressibility factor) is dimensionless, \( R \) (gas constant) is in \( \text{psia ft}^3/\text{lbm mol °R} \).

Molecular weight and the gas constant are constants that have negligible uncertainties. Therefore, the uncertainty in gas density can be calculated by:

\[ \frac{\delta \rho}{\rho} = \left( \frac{\delta P}{P} \right)^2 + \left( \frac{\delta T}{T} \right)^2 + \left( \frac{\delta Z}{Z} \right)^2 \]

Compressibility factor describes the deviation of a real gas from that of an ideal gas and is derived from an equation of state. A 0.1% uncertainty was assumed for the compressibility factor and was used in the calculations. Pressure and temperature are measured values and therefore have the uncertainty of the transmitter. Conservative estimates for uncertainty of pressure and temperature measurements are 0.075% and 0.2%, respectively. Therefore the gas density uncertainty was calculated to be 0.236%.

Liquid density is a function of temperature. Therefore to obtain the liquid density uncertainty, the uncertainties in temperature and an equation of state are needed. Using the conservative estimate of 0.2% uncertainty for temperature and assuming an uncertainty of 0.1% in the equation of state results in calculated density uncertainty of 0.224%.

Differential Pressure Uncertainty

Differential pressure, like pressure and temperature, is a measured value. Therefore the uncertainty is that of the transmitter. A conservative estimate of the uncertainty is 0.075% for full range conditions.

Calculation of Flow Uncertainty

Assumptions:
Differential Pressure=150 inches of water
Pressure=200 psia
Meter Tub Bore=2 inches
\( \beta=0.6 \)

<table>
<thead>
<tr>
<th>Table 1.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gas</strong></td>
</tr>
<tr>
<td>Cd</td>
</tr>
<tr>
<td>Y1</td>
</tr>
<tr>
<td>D</td>
</tr>
<tr>
<td>d</td>
</tr>
<tr>
<td>h</td>
</tr>
<tr>
<td>p</td>
</tr>
<tr>
<td>Total</td>
</tr>
</tbody>
</table>

The column labeled % Cont. shows the relative contributions of each term to the overall flow uncertainty. In both cases, the discharge coefficient proves to be the largest contributor to the overall uncertainty, while differential pressure is the smallest contributor.
Orifice Meter Rangeability

The Multivariable™ transmitters measure differential pressure, pressure, and temperature. To calculate flow uncertainties actual differential pressure data was used. The test results showed the average pressure uncertainty to be 0.04%. The temperature uncertainty was assumed to be 0.2%. The units tested had 250 inches of water URL (Upper Range Limit) differential pressure sensors, and 800 psia URL pressure sensors. The sensors for each unit were trimmed at zero and URL prior to testing. Figure 2 shows the differential pressure range down results at 200 psia static pressure along with the calculated flow uncertainty for the air flow example. The results for the water flow example were essentially the same.

FIGURE 2. DP Error and Flow Uncertainty vs. Flow Range

The affect on flow uncertainty from the differential pressure error as flow is reduced is clearly demonstrated by Figure 2. Differential pressure does not significantly affect the flow uncertainty until approximately an 8:1 range down in flow is achieved. Table 2 shows a breakdown of the maximum flow uncertainty. This point occurred at 100:1 range down in differential pressure or 10:1 in flow.

Table 2.

<table>
<thead>
<tr>
<th>Term</th>
<th>Uncertainty %</th>
<th>% Cont.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cd</td>
<td>0.6</td>
<td>46.26</td>
</tr>
<tr>
<td>Y1</td>
<td>0.108</td>
<td>1.50</td>
</tr>
<tr>
<td>D</td>
<td>0.4</td>
<td>1.82</td>
</tr>
<tr>
<td>d</td>
<td>0.07</td>
<td>3.32</td>
</tr>
<tr>
<td>h</td>
<td>1.19</td>
<td>45.53</td>
</tr>
<tr>
<td>ρ</td>
<td>0.224</td>
<td>1.66</td>
</tr>
<tr>
<td>Total</td>
<td>0.88</td>
<td>100</td>
</tr>
</tbody>
</table>

Even at high range downs, exceeding 8:1 range down in flow, the overall calculated flow uncertainty was less than 1% for all four transmitters.

As stated earlier, common practice is to simplify the flow equation which results in larger flow uncertainties. Figure 3 shows the flow uncertainty using the simplified flow equations for the air and water examples examined.

FIGURE 3. Flow Uncertainty using Simplified Flow Equations vs. Flow Range

A comparison of Figure 2 and Figure 3 illustrates the improvement that can be obtained by using the fully compensated flow equation.

CONCLUSION

It has been assumed in the past that the performance of the instrumentation used with orifice meters limits the useful range to 3:1 in flow. It has been shown here that the current state of the art instrumentation can extend the rangeability to approximately 8:1. The performance of typical Multivariable transmitters has been demonstrated to be capable of such flow rangeabilities. It has also been shown that parameters related to the physical installation, such as the orifice meter dimensions and discharge coefficient, and parameters related to the fluid properties, such as fluid density and viscosity, contribute significantly to orifice meter uncertainty. Multiple process variable transmitters with the capability of continuous calculation of these parameters will produce the maximum benefit for user by providing accurate, field-based mass flow measurement over a much wider range than has been achievable in the past.
REFERENCES


3. AIChE: Data Compilation Table of Properties of Pure Compounds, Design Institute for Physical Property Data, American Institute Chemical Engineers, New York, 1986


11. AIChE, ibid.